Lesson-by-Lesson Review

• Complete the highlighted problems.

Trigonometric Identities (pp. 312–319)

Find the value of each expression using the given information.

11. $\sec\theta$ and $\cos\theta$; $\tan\theta = 3$, $\cos\theta > 0$
12. $\cot\theta$ and $\sin\theta$; $\cos\theta = -\frac{1}{5}$, $\tan\theta < 0$
13. $\csc\theta$ and $\tan\theta$; $\cos\theta = \frac{3}{5}$, $\sin\theta < 0$
14. $\cot\theta$ and $\cos\theta$; $\tan\theta = \frac{2}{7}$, $\csc\theta > 0$
15. $\sec\theta$ and $\sin\theta$; $\cot\theta = -2$, $\csc\theta < 0$
16. $\cos\theta$ and $\sin\theta$; $\cot\theta = \frac{3}{8}$, $\sec\theta < 0$

Simplify each expression.

17. $\sin^2(-x) + \cos^2(-x)$
18. $\sin^2 x + \cos^2 x + \cot^2 x$
19. $\frac{\sec^2 x - \tan^2 x}{\cos(-x)}$
20. $\frac{\sec^2 x}{\tan^2 x + 1}$
21. $\frac{1}{1 - \sin x}$
22. $\frac{\cos x}{1 + \sec x}$

Example 1

If $\sec\theta = -3$ and $\sin\theta > 0$, find $\sin\theta$.

Since $\sin\theta > 0$ and $\sec\theta < 0$, θ must be in Quadrant II. To find $\sin\theta$, first find $\cos\theta$ using the Reciprocal Identity for $\sec\theta$ and $\cos\theta$.

$\cos\theta = \frac{1}{\sec\theta}$ Reciprocal Identity

$= -\frac{1}{3}$ $\sec\theta = -3$

Now you can use the Pythagorean identity that includes $\sin\theta$ and $\cos\theta$ to find $\sin\theta$.

$\sin^2\theta + \cos^2\theta = 1$ Pythagorean Identity

$\sin^2\theta + \left(-\frac{1}{3}\right)^2 = 1$ $\cos\theta = -\frac{1}{3}$

$\sin^2\theta + \frac{1}{9} = 1$ Multiply.

$\sin^2\theta = \frac{8}{9}$ Subtract.

$\sin\theta = \frac{\sqrt{8}}{3}$ or $\frac{2\sqrt{2}}{3}$ Simplify.

5-2 Verifying Trigonometric Identities (pp. 320–326)

Verify each identity.

23. $\frac{\sin\theta}{1 - \cos\theta} + \frac{\sin\theta}{1 + \cos\theta} = 2\csc\theta$
24. $\frac{\cos\theta}{\sec\theta} + \frac{\sin\theta}{\csc\theta} = 1$
25. $\frac{\cot\theta}{1 + \csc\theta} + \frac{1 + \csc\theta}{\cot\theta} = 2\sec\theta$
26. $\frac{\cos\theta}{1 - \sin\theta} = \frac{1 + \sin\theta}{\cos\theta}$
27. $\frac{\cot^2\theta}{1 + \csc\theta} = \csc\theta - 1$
28. $\frac{\sec\theta}{\tan\theta} + \frac{\csc\theta}{\cot\theta} = \sec\theta + \csc\theta$
29. $\frac{\sec\theta + \csc\theta}{1 + \tan\theta} = \csc\theta$
30. $\cot\theta\csc\theta + \sec\theta = \csc^2\theta\sec\theta$
31. $\frac{\sin\theta}{\sin\theta + \cos\theta} = \frac{\tan\theta}{1 + \tan\theta}$
32. $\cos^4\theta - \sin^4\theta = \frac{1 - \tan^2\theta}{\sec^2\theta}$

Example 2

Verify that $\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} = 2\csc\theta$.

The left-hand side of this identity is more complicated, so start with that expression.

$$\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1 + \cos\theta)^2}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{1 + 1 + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{2 + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{2(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\csc\theta$$

Lesson-by-Lesson Review

5-3 Solving Trigonometric Equations (pp. 327–333)

Find all solutions of each equation on the interval [0 , 2pi) ~~$[0, 2\pi]$~~.

33. $2 \sin x = \sqrt{2}$

34. $4 \cos^2 x = 3$

35. $\tan^2 x - 3 = 0$

36. $9 + \cot^2 x = 12$

37. $2 \sin^2 x = \sin x$

38. $3 \cos x + 3 = \sin^2 x$

Solve each equation for all values of *x*.

39. $\sin^2 x - \sin x = 0$

40. $\tan^2 x = \tan x$

41. $3 \cos x = \cos x - 1$

42. $\sin^2 x = \sin x + 2$

43. $\sin^2 x = 1 - \cos x$

44. $\sin x = \cos x + 1$

Example 3

Solve the equation $\sin \theta = 1 - \cos \theta$ for all values of θ.

$\sin \theta = 1 - \cos \theta$	Original equation.
$\sin^2 \theta = (1 - \cos \theta)^2$	Square each side.
$\sin^2 \theta = 1 - 2 \cos \theta + \cos^2 \theta$	Expand.
$1 - \cos^2 \theta = 1 - 2 \cos \theta + \cos^2 \theta$	Pythagorean Identity
$0 = 2 \cos^2 \theta - 2 \cos \theta$	Subtract.
$0 = 2 \cos \theta(\cos \theta - 1)$	Factor.

Solve for x on $[0, 2\pi]$.

$\cos \theta = 0$ or $\cos \theta = 1$

$\theta = \cos^{-1} 0$ $\quad$ $\theta = \cos^{-1} 1$

$\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ $\quad$ $\theta = 0$

A check shows that $\frac{3\pi}{2}$ is an extraneous solution. So the solutions are $\theta = \frac{\pi}{2} + 2n\pi$ or $\theta = 0 + 2n\pi$.

5-4 Sum and Difference Identities (pp. 336–343)

Find the exact value of each trigonometric expression.

45. $\cos 15°$

46. $\sin 345°$

47. $\tan \frac{13\pi}{12}$

48. $\sin \frac{7\pi}{12}$

49. $\cos -\frac{11\pi}{12}$

50. $\tan \frac{5\pi}{12}$

Simplify each expression.

51. $\dfrac{\tan \frac{\pi}{9} + \tan \frac{8\pi}{9}}{1 - \tan \frac{\pi}{9} \tan \frac{8\pi}{9}}$

52. $\cos 24° \cos 36° - \sin 24° \sin 36°$

53. $\sin 95° \cos 50° - \cos 95° \sin 50°$

54. $\cos \frac{2\pi}{9} \cos \frac{\pi}{18} + \sin \frac{2\pi}{9} \sin \frac{\pi}{18}$

Verify each identity.

55. $\cos (\theta + 30°) - \sin (\theta + 60°) = -\sin \theta$

56. $\cos \left(\theta + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} (\cos \theta - \sin \theta)$

57. $\cos \left(\theta - \frac{\pi}{3}\right) + \cos \left(\theta + \frac{\pi}{3}\right) = \cos \theta$

58. $\tan \left(\theta + \frac{3\pi}{4}\right) = \frac{\tan \theta - 1}{\tan \theta + 1}$

Example 4

Find the exact value of $\tan \frac{23\pi}{12}$.

$\tan \frac{23\pi}{12} = \tan \left(\frac{5\pi}{4} + \frac{2\pi}{3}\right)$	$\frac{23\pi}{12} = \frac{5\pi}{4} + \frac{2\pi}{3}$
$= \dfrac{\tan \frac{5\pi}{4} + \tan \frac{2\pi}{3}}{1 - \tan \frac{5\pi}{4} \tan \frac{2\pi}{3}}$	Sum Identity
$= \dfrac{1 - \sqrt{3}}{1 - (-\sqrt{3})}$	Evaluate for tangent.
$= \dfrac{1 - \sqrt{3}}{1 + \sqrt{3}}$	Simplify.
$= \dfrac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \dfrac{1 - \sqrt{3}}{1 - \sqrt{3}}$	Rationalize the denominator.
$= \dfrac{4 - 2\sqrt{3}}{1 - 3}$	Multiply.
$= \dfrac{4 - 2\sqrt{3}}{-2}$ or $-2 + \sqrt{3}$	Simplify.

Lesson-by-Lesson Review

5-5 Multiple-Angle and Product-Sum Identities (pp. 346–354)

Find the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ for the given value and interval.

59. $\cos \theta = \frac{1}{3}, (0°, 90°)$

60. $\tan \theta = 2, (180°, 270°)$

61. $\sin \theta = \frac{4}{5}, \left(\frac{\pi}{2}, \pi\right)$

62. $\sec \theta = \frac{13}{5}, \left(\frac{3\pi}{2}, 2\pi\right)$

Find the exact value of each expression. (Half Angle Formulas)

63. $\sin 75°$

64. $\cos \frac{11\pi}{12}$

65. $\tan 67.5°$

66. $\cos \frac{3\pi}{8}$

67. $\sin \frac{15\pi}{8}$

68. $\tan \frac{13\pi}{12}$

Example 5

Find the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ if θ is in the fourth quadrant and $\tan \theta = -\frac{24}{7}$.

θ is in the fourth quadrant, so $\cos \theta = \frac{7}{25}$ and $\sin \theta = -\frac{24}{25}$.

$\sin 2\theta = 2 \sin \theta \cos \theta$

$= 2\left(-\frac{24}{25}\right)\frac{7}{25}$ or $-\frac{336}{625}$

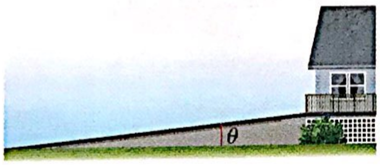
$\cos 2\theta = 2\cos^2 \theta - 1$

$= 2\left(\frac{7}{25}\right)^2 - 1$ or $-\frac{527}{625}$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2\left(-\frac{24}{7}\right)}{1 - \left(-\frac{24}{7}\right)^2} = \frac{-\frac{48}{7}}{-\frac{527}{49}} \text{ or } \frac{336}{527}$$

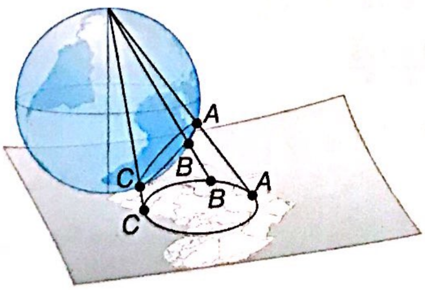
Applications and Problem Solving

69. **CONSTRUCTION** Find the tangent of the angle that the ramp makes with the building if $\sin \theta = \frac{\sqrt{145}}{145}$ and $\cos \theta = \frac{12\sqrt{145}}{145}$. (Lesson 5-1)



70. **LIGHT** The intensity of light that emerges from a system of two polarizing lenses can be calculated by $I = I_0 - \frac{I_0}{\csc^2 \theta}$, where I_0 is the intensity of light entering the system and θ is the angle of the axis of the second lens with the first lens. Write the equation for the light intensity using only $\tan \theta$. (Lesson 5-1)

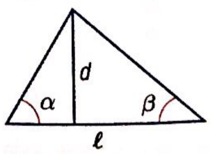
71. **MAP PROJECTIONS** Stereographic projection is used to project the contours of a three-dimensional sphere onto a two-dimensional map. Points on the sphere are related to points on the map using $r = \frac{\sin \alpha}{1 - \cos \alpha}$. Verify that $r = \frac{1 + \cos \alpha}{\sin \alpha}$. (Lesson 5-2)



72. **PROJECTILE MOTION** A ball thrown with an initial speed v_0 at an angle θ that travels a horizontal distance d will remain in the air t seconds, where $t = \frac{d}{v_0 \cos \theta}$. Suppose a ball is thrown with an initial speed of 50 feet per second, travels 100 feet, and is in the air for 4 seconds. Find the angle at which the ball was thrown. (Lesson 5-3)

73. **BROADCASTING** Interference occurs when two waves pass through the same space at the same time. It is destructive if the amplitude of the sum of the waves is less than the amplitudes of the individual waves. Determine whether the interference is destructive when signals modeled by $y = 20 \sin (3t + 45°)$ and $y = 20 \sin (3t + 225°)$ are combined. (Lesson 5-4)

74. **TRIANGULATION** *Triangulation* is the process of measuring a distance d using the angles α and β and the distance ℓ using $\ell = \frac{d}{\tan \alpha} + \frac{d}{\tan \beta}$. (Lesson 5-5)



a. Solve the formula for d.

b. Verify that $d = \frac{\ell \sin \alpha \sin \beta}{\sin \alpha \cos \beta + \cos \alpha \sin \beta}$.

c. Verify that $d = \frac{\ell \sin \alpha \sin \beta}{\sin (\alpha + \beta)}$.

d. Show that if $\alpha = \beta$, then $d = 0.5\ell \tan \alpha$.