Name Key

Unit 1: Matrices Test Study Guide

True/False—

F 1. The identity matrix is always a 3 x 3 matrix. Can be any square matrix

T 2. Every matrix can be multiplied by a scalar (number) by multiplying each entry by that number.

F 3. A 3 x 5 matrix can be an identity matrix if it has 1s on the main diagonal and 0s for all the other entries. only square matrix

F 4. A square matrix can be inverted if its determinant is 0.

F 5. If A and B are both 4 x 5 matrices then A and B can be added and subtracted in any order. $A+B=B+A$ but $A-B \neq B-A$

F 6. If A and B are both 6 x 2 matrices then AB=BA. $6\times 2 \times 6 \times 2$ can't multiply

T 7. If $A=\begin{bmatrix}1 & 2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}1\\2\\3\end{bmatrix}$ then $\cancel{BA}$ AB $=[14]$. $\begin{bmatrix}1\\2\\3\end{bmatrix}\cdot\begin{bmatrix}1 & 2 & 3\end{bmatrix} = [1+4+9] = [14]$

F 8. AI =A for all matrices A and the appropriate identity matrix I. *only for square matrices

ex: $\begin{bmatrix}2 & 3\\-4 & 5\end{bmatrix}\cdot\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}2 & 3\\-4 & 5\end{bmatrix}$

F 9. $\begin{vmatrix}5 & -3\\2 & 1\end{vmatrix} = -1$

det $= 5-(-6) = 11 \neq -1$

For #10 – 17, given $A=\begin{bmatrix}2 & -1 & 3\\4 & 1 & -9\end{bmatrix}$, $B=\begin{bmatrix}-6 & 0 & 3\\-5 & -1 & 0\end{bmatrix}$, $C=\begin{bmatrix}-4 & 3\\1 & 5\end{bmatrix}$, and $D=\begin{bmatrix}-1 & 3 & 0\\8 & 1 & -2\\11 & 0 & 3\end{bmatrix}$

10. The order (dimension) of matrix A is:

A. 6 B. 3 x 2 C. 2 x 3 D. 6 x 1

11. $G=\begin{bmatrix}-4\\2\\3\end{bmatrix}$, which of the following is possible.

A. GA B. BG C. G + B D. GD

$(2\times 3)(3\times 1)$

12. B – A = ?

$$\begin{bmatrix}-6 & 0 & 3\\-5 & -1 & 0\end{bmatrix} - \begin{bmatrix}2 & -1 & 3\\4 & 1 & -9\end{bmatrix} = \begin{bmatrix}-8 & 1 & 0\\-9 & -2 & 9\end{bmatrix}$$

13. CB = ?

$$\begin{bmatrix}-4 & 3\\1 & 5\end{bmatrix}\cdot\begin{bmatrix}-6 & 0 & 3\\-5 & -1 & 0\end{bmatrix} = \begin{bmatrix}24+-15 & 0+-3 & -12+0\\-6+-25 & 0+-5 & 3+0\end{bmatrix} = \begin{bmatrix}9 & -3 & -12\\-31 & -5 & 3\end{bmatrix}$$

2×2 2×3 2×3

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14. 3D =?

$3\begin{bmatrix}-1 & 3 & 0\\ 8 & 1 & -2\\ 11 & 0 & 3\end{bmatrix} = \begin{bmatrix}-3 & 9 & 0\\ 24 & 3 & -6\\ 33 & 0 & 9\end{bmatrix}$

15. det D =?

$\begin{vmatrix}-1 & 3 & 0 & -1 & 3\\ 8 & 1 & -2 & 8 & 1\\ 11 & 0 & 3 & 11 & 0\end{vmatrix} = (-3 + -66 + 0) - (0 + 0 + 72) = -141$

16. -5A=?

$-5\begin{bmatrix}2 & -1 & 3\\ 4 & 1 & -9\end{bmatrix} = \begin{bmatrix}-10 & 5 & -15\\ -20 & -5 & 45\end{bmatrix}$

17. det C=?

$\begin{vmatrix}-4 & 3\\ 1 & 5\end{vmatrix} = -20-3 = -23$

19. If $A=\begin{bmatrix}1 & -2 & -4\end{bmatrix}$ and $B=\begin{bmatrix}-9\\ 0\\ 10\end{bmatrix}$ find AB.

$\underset{1\times 3}{\begin{bmatrix}1 & -2 & -4\end{bmatrix}}\underset{3\times 1}{\begin{bmatrix}-9\\ 0\\ 10\end{bmatrix}} = \underset{1\times 1}{\begin{bmatrix}-9+0+-40\end{bmatrix}} = \begin{bmatrix}-49\end{bmatrix}$

20. Determine if matrix $A=\begin{bmatrix}3 & 6\\ -2 & -4\end{bmatrix}$ has an inverse. Explain why or why not.

det = -12 - -12 = 0 ⇒ NO b/c det = 0

21. Find A^{-1}, if it exists, for $A=\begin{bmatrix}-2 & 7\\ 1 & -4\end{bmatrix}$. Show work.

$A^{-1} = \frac{1}{8-7}\begin{bmatrix}-4 & -7\\ -1 & -2\end{bmatrix} = \frac{1}{1}\begin{bmatrix}-4 & -7\\ -1 & -2\end{bmatrix} = \begin{bmatrix}-4 & -7\\ -1 & -2\end{bmatrix}$

22. Given A = B, find x, y and z. $A=\begin{bmatrix}2x-1 & 4 & -7\\ -3 & 5y & 2y+1\\ 0 & -3 & z\end{bmatrix}$ $B=\begin{bmatrix}27 & 4 & -\frac{1}{2}x\\ -3 & 10 & 5\\ 0 & -3 & 4z\end{bmatrix}$. Show all work as needed.

$2x-1=27$
$+1 \quad +1$
$2x=28$
$\boxed{x=14}$

$\frac{5y}{5}=\frac{10}{5}$
$\boxed{y=2}$

$z=4z$
$-z \quad -z$
$0=3z$
$\boxed{0=z}$

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23. Solve for x: $\underbrace{\overset{2\times 3}{\begin{bmatrix} 1 & 3x & 2 \\ 3 & 0 & 5 \end{bmatrix}}\overset{3\times 2}{\begin{bmatrix} 4 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix}}} - \begin{bmatrix} 16 & 3 \\ 6 & 2 \end{bmatrix} = 3\begin{bmatrix} x-2 & 0 \\ 2 & 2 \end{bmatrix}$

$\begin{bmatrix} 4+6x+0 & 1+0+2 \\ 12+0+0 & 3+0+5 \end{bmatrix} \cdot \begin{bmatrix} 4+6x & 3 \\ 12 & 8 \end{bmatrix} - \begin{bmatrix} 16 & 3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 3x-6 & 0 \\ 6 & 6 \end{bmatrix}$

$4+6x-16=3x-6$

$6x-12=3x-6$

$-3x \quad -3x$

$3x-12=-6$

$3x=6$

$\boxed{x=2}$

24. Solve for x: $\begin{vmatrix} 12 & 4 & -6 \\ 2 & x & 3 \\ x & 0 & -2 \end{vmatrix} = 6x+4$

$\begin{vmatrix} 12 & 4 & -6 \\ 2 & x & 3 \\ x & 0 & -2 \end{vmatrix}\begin{matrix} 12 & 4 \\ 2 & x \\ x & 0 \end{matrix} = (-24x+12x+0)-(-6x^2+0-16)$

$-12x-(-6x^2-16)$

$-12x+6x^2+16$

$6x^2-12x+16=6x+4$

$\dfrac{6x^2-18x+12=0}{6}$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$\boxed{\begin{matrix} x=2 \\ x=1 \end{matrix}}$

25. Use the inverse matrix to solve the system of equations.

$\begin{cases} 6x+5y=13 \\ 2x+2y=5 \end{cases}$

$\underbrace{\begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$

inverse

$\dfrac{1}{12-10}\begin{bmatrix} 2 & -5 \\ \cancel{-2} & 6 \end{bmatrix} = \dfrac{1}{2}\begin{bmatrix} 2 & -5 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -5/2 \\ -1 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & -5/2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 13 \\ 5 \end{bmatrix} = \begin{bmatrix} 13-\frac{25}{2} \\ -13+15 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 \\ 2 \end{bmatrix}$

26. Write the matrix equation associated with the system of equations and solve (may use calc., but show steps)

$\begin{cases} x+y=3 \\ -x+3y+4z=-3 \\ 4y+3z=2 \end{cases}$

$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7/4 & 3/4 & -1 \\ -3/4 & -3/4 & 1 \\ 1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

27. Write the matrix equation associated with the system of equations and solve (may use calc., but show steps)

$\begin{cases} 2x+y-z=3 \\ -x+2y+4z=-3 \\ x-2y-3z=4 \end{cases}$

$\begin{bmatrix} 2 & 1 & -1 \\ -1 & 2 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2/5 & 1 & 6/5 \\ 1/5 & -1 & -7/5 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

28. Write a system of equations for the following situation and solve using a matrix equation.

B = # of BD ballons C = # of congrats G = # of get well balloons

You have $33 to spend on 24 balloons. Birthday balloons cost $1.50 each, congratulations balloons cost $1.00 each, and get well balloons cost $2.00 each. You want twice as many birthday balloons as the other two types combined. Calculate how many of each you should buy.

$$\begin{cases} B + C + G = 24 \\ 1.5B + C + 2G = 33 \\ B - 2C - 2G = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1.5 & 1 & 2 \\ 1 & -2 & -2 \end{bmatrix}\begin{bmatrix} B \\ C \\ G \end{bmatrix} = \begin{bmatrix} 24 \\ 33 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} B \\ C \\ G \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.5 & 1 & 2 \\ 1 & -2 & -2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 24 \\ 33 \\ 0 \end{bmatrix}$$

$$B = 2(C+G) \Rightarrow B = 2C + 2G \Rightarrow B - 2C - 2G = 0$$

$$\begin{bmatrix} B \\ C \\ G \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 1 \end{bmatrix}$$

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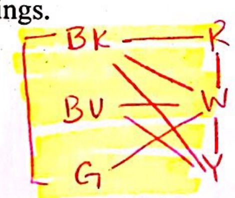
29. Your cross country team is ordering uniforms. Each person must have a practice uniform and a competition uniform. Some stipulations have been placed on the selection of colors. Blue can only pair with white or yellow. Yellow can pair with black, white, or blue. Red can pair with black or white. Black can pair with green, white, yellow or red. Green can pair with black or white. White can pair with all colors except itself.

a. Draw a graph to represent the color pairings.



b. Create a matrix to represent the graph.

*will be different depending on how you labeled

$$\begin{array}{c|c} & \begin{matrix} BK & BU & G & R & W & Y \end{matrix} \\ \begin{matrix} BK \\ BU \\ G \\ R \\ W \\ Y \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

30. **Encode** the following messages using $\begin{bmatrix} 1 & 9 & 6 \\ 5 & -2 & 1 \\ 2 & 1 & -3 \end{bmatrix}$ as your encryption matrix and then **decode** it back to the original message:

Today is Friday

20 15 4 1 25 0 9 19 0 6 18 9 4 1 25

$$\underbrace{\begin{bmatrix} 20 & 15 & 4 \\ 1 & 25 & 0 \\ 9 & 19 & 0 \\ 6 & 18 & 9 \\ 4 & 1 & 25 \end{bmatrix}}_{\text{message}} \underbrace{\begin{bmatrix} 1 & 9 & 6 \\ 5 & -2 & 1 \\ 2 & 1 & -3 \end{bmatrix}}_{\text{encrypt matrix}} = \underbrace{\begin{bmatrix} 103 & 154 & 123 \\ 126 & -41 & 31 \\ 104 & 43 & 73 \\ 114 & 27 & 27 \\ 59 & 59 & -50 \end{bmatrix}}_{\text{coded matrix}}$$

103 154 123 126 -41 31 104 43 73 114 27 27 59 59 -50

$$\begin{bmatrix} 103 & 154 & 123 \\ 126 & -41 & 31 \\ 104 & 43 & 73 \\ 114 & 27 & 27 \\ 59 & 59 & -50 \end{bmatrix} \cdot \begin{bmatrix} 1 & 9 & 6 \\ 5 & -2 & 1 \\ 2 & 1 & -3 \end{bmatrix}^{-1} = \begin{bmatrix} 20 & 15 & 4 \\ 1 & 25 & 0 \\ 9 & 19 & 0 \\ 6 & 18 & 9 \\ 4 & 1 & 25 \end{bmatrix}$$

T	O	D	A	Y	I	S
20	15	4	1	25	9	19

F	R	I	D	A	Y
6	18	9	4	1	25