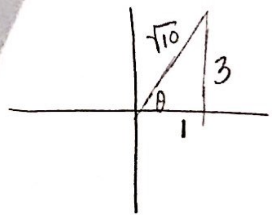
{Unit 3D Review}

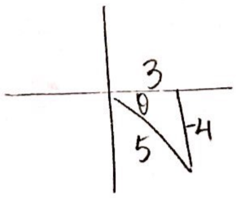
(11) $\tan\theta = 3$, $\cos\theta > 0$



$$* \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{\sqrt{10}}} = \boxed{\sqrt{10}}$$

$$* \cos\theta = \frac{1}{\sqrt{10}} = \boxed{\frac{\sqrt{10}}{10}}$$

(13) $\cos\theta = \frac{3}{5}$, $\sin\theta < 0$



$$* \csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{4}{5}} = \boxed{\frac{-5}{4}}$$

$$* \tan\theta = \boxed{\frac{-4}{3}}$$

(17) $\sin^2(-x) + \cos^2(-x) = -\sin^2 x + \underbrace{\cos^2 x}_{1-\sin^2 x} = -\sin^2 x + 1 - \sin^2 x = 1 - 2\sin^2 x$

(19) $$\frac{\overbrace{\sec^2 x}^{1+\tan^2 x} - \tan^2 x}{\cos(-x)} = \frac{1 + \tan^2 x - \tan^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$$

(21) $$\frac{1}{1-\sin x} = \frac{1}{(1-\sin x)} \cdot \frac{(1+\sin x)}{(1+\sin x)} = \frac{1+\sin x}{1-\sin^2 x} = \frac{1+\sin x}{\cos^2 x} = \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$$

$$= \sec^2 x + \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sec^2 x + \tan x \sec x$$

(23) $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\csc\theta$

$\frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} + \frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} \to \frac{\sin\theta + \cancel{\sin\theta\cos\theta} + \sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta}$

$\to \frac{2\cancel{\sin\theta}}{\sin^{\cancel{2}}\theta} \to \frac{2}{\sin\theta} = 2\csc\theta$

(25) $\frac{\cot\theta}{1+\csc\theta} + \frac{1+\csc\theta}{\cot\theta} = 2\sec\theta$

Attempt #1 $\frac{\cot\theta}{(1+\csc\theta)} \cdot \frac{\cot\theta}{\cot\theta} + \frac{(1+\csc\theta)(1+\csc\theta)}{\cot\theta(1+\csc\theta)} \to \frac{\cot^2\theta + 1 + 2\csc\theta + \overbrace{\csc^2\theta}^{1+\cot^2\theta}}{\cot\theta(1+\csc\theta)}$

$\to \frac{2+2\cot^2\theta + 2\csc\theta}{\cot\theta(1+\csc\theta)}$ ... ugh!! :(

Attempt #2 $\frac{\frac{\cos\theta}{\sin\theta}}{1+\frac{1}{\sin\theta}} + \frac{1+\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} \to \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta+1}{\sin\theta}} + \frac{\frac{\sin\theta+1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} \to \frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{\sin\theta+1} + \frac{\sin\theta+1}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{\cos\theta}$

$\to \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} \to \frac{\cos\theta}{(1+\sin\theta)} \cdot \frac{\cos\theta}{\cos\theta} + \frac{(1+\sin\theta)}{\cos\theta} \cdot \frac{(1+\sin\theta)}{(1+\sin\theta)}$

$\to \frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)} \to \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} \to \frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}} \to \frac{2}{\cos\theta} = 2\sec\theta$:)

$$\frac{\cot^2\theta}{1+\csc\theta} = \csc\theta - 1$$

$$\frac{\cot^2\theta}{(1+\csc\theta)} \cdot \frac{(1-\csc\theta)}{(1-\csc\theta)} \rightarrow \frac{\cot^2\theta(1-\csc\theta)}{1-\underbrace{\csc^2\theta}_{1+\cot^2\theta}} \rightarrow \frac{\cot^2\theta(1-\csc\theta)}{1-(1+\cot^2\theta)}$$

$$\rightarrow \frac{\cancel{\cot^2\theta}(1-\csc\theta)}{-\cancel{\cot^2\theta}} \rightarrow \frac{(1-\csc\theta)}{-1} \rightarrow -1+\csc\theta = \csc\theta - 1$$

(29) $$\frac{\sec\theta + \csc\theta}{1+\tan\theta} = \csc\theta$$

$$\frac{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}{1+\frac{\sin\theta}{\cos\theta}} \rightarrow \frac{\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}} \rightarrow \frac{\cancel{\sin\theta+\cos\theta}}{\cos\theta\sin\theta} \cdot \frac{\cos\theta}{\cancel{\cos\theta+\sin\theta}} \rightarrow \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}\sin\theta} \rightarrow \frac{1}{\sin\theta} = \csc\theta$$

(31) $$\frac{\sin\theta}{\sin\theta+\cos\theta} = \frac{\tan\theta}{1+\tan\theta}$$

$$\frac{\sin\theta}{\sin\theta+\cos\theta} \cdot \frac{\left(\frac{1}{\cos\theta}\right)}{\left(\frac{1}{\cos\theta}\right)} \rightarrow \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta}} \rightarrow \frac{\tan\theta}{\tan\theta+1} = \frac{\tan\theta}{1+\tan\theta}$$

(33) $2\sin x = \sqrt{2}$; $[0, 2\pi)$

$\sin x = \frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}}$

(39) $\sin^2 x - \sin x = 0$ for all

$\sin x(\sin x - 1) = 0$

$\sin x = 0 \quad \sin x - 1 = 0$

$x = 0 + n\pi \quad \sin x = 1$

$\boxed{x = n\pi} \quad \boxed{x = \frac{\pi}{2} + 2n\pi}$

(35) $\tan^2 x - 3 = 0$; $[0, 2\pi)$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

(41) $3\cos x = \cos x - 1$ for all x

$2\cos x = -1$

$\cos x = \frac{-1}{2}$

$\boxed{x = \frac{2\pi}{3} + 2n\pi, \quad \frac{4\pi}{3} + 2n\pi}$

(37) $2\sin^2 x = \sin x$; $[0, 2\pi)$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0 \quad 2\sin x - 1 = 0$

$\boxed{x = 0, \pi} \quad \sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

(43) $\underbrace{\sin^2 x}_{\text{Pythagorean}} = 1 - \cos x$ for all x

$1 - \cos^2 x = 1 - \cos x$

$0 = \cos^2 x - \cos x$

$0 = \cos x(\cos x - 1)$

$\cos x = 0 \quad \cos x - 1 = 0$

$\boxed{x = \frac{\pi}{2} + n\pi} \quad \cos x = 1$

$x = 0 + 2n\pi$

$\boxed{x = 2n\pi}$

Sum + Difference Formulas

(45) $\cos 15° = \cos(60° - 45°) = \cos 60° \cos 45° + \sin 60° \sin 45°$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$

(47) $\tan\frac{13\pi}{12} = \tan\left(\frac{\pi}{3} + \frac{3\pi}{4}\right) = \frac{\tan\frac{\pi}{3} + \tan\frac{3\pi}{4}}{1 - \tan\frac{\pi}{3} \cdot \tan\frac{3\pi}{4}} = \frac{\sqrt{3} + -1}{1 - \sqrt{3} \cdot -1} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{(\sqrt{3} - 1)(1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})}$

$\frac{4}{12} + \frac{9}{12}$

$\frac{\pi}{3} \quad \frac{3\pi}{4}$

$= \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - 3} = \frac{2\sqrt{3} - 4}{-2} = \boxed{-\sqrt{3} + 2}$

Scanned with CamScanner

$$\frac{\tan\frac{\pi}{9}+\tan\frac{8\pi}{9}}{1-\tan\frac{\pi}{9}\tan\frac{8\pi}{9}} = \tan\left(\frac{\pi}{9}+\frac{8\pi}{9}\right) = \tan\pi = 0$$

(53) $\sin 95^\circ \cos 50^\circ - \cos 95^\circ \sin 50^\circ = \sin(95^\circ - 50^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$

(55) $\underbrace{\cos(\theta+30^\circ)} - \underbrace{\sin(\theta+60^\circ)} = -\sin\theta$

$$\left(\cos\theta\underbrace{\cos 30^\circ}_{\frac{\sqrt{3}}{2}} - \sin\theta\underbrace{\sin 30^\circ}_{\frac{1}{2}}\right) - \left(\sin\theta\underbrace{\cos 60^\circ}_{\frac{1}{2}} + \cos\theta\underbrace{\sin 60^\circ}_{\frac{\sqrt{3}}{2}}\right)$$

$$\cancel{\frac{\sqrt{3}}{2}\cos\theta}\underbrace{-\frac{1}{2}\sin\theta - \frac{1}{2}\sin\theta}_{-\sin\theta\ ✓}\cancel{-\frac{\sqrt{3}}{2}\cos\theta}$$

(57) $\underbrace{\cos\left(\theta-\frac{\pi}{3}\right)} + \underbrace{\cos\left(\theta+\frac{\pi}{3}\right)} = \cos\theta$

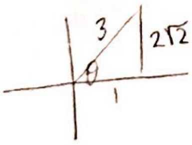
$$\left(\cos\theta\underbrace{\cos\frac{\pi}{3}}_{\frac{1}{2}} + \sin\theta\underbrace{\sin\frac{\pi}{3}}_{\frac{\sqrt{3}}{2}}\right) + \left(\cos\theta\underbrace{\cos\frac{\pi}{3}}_{\frac{1}{2}} - \sin\theta\underbrace{\sin\frac{\pi}{3}}_{\frac{\sqrt{3}}{2}}\right)$$

$$\frac{1}{2}\cos\theta + \cancel{\frac{\sqrt{3}}{2}\sin\theta} + \frac{1}{2}\cos\theta - \cancel{\frac{\sqrt{3}}{2}\sin\theta}$$

$$\frac{1}{2}\cos\theta + \frac{1}{2}\cos\theta = \cos\theta\ ✓$$

Double Angle Formulas

(59) $\cos\theta = \frac{1}{3}$, $(0^\circ, 90^\circ)$

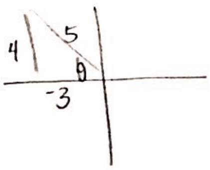


* $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{2\sqrt{2}}{3}\cdot\frac{1}{3} = \boxed{\frac{4\sqrt{2}}{9}}$

* $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{1}{3}\right)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2 = \frac{1}{9} - \frac{8}{9} = \boxed{\frac{-7}{9}}$

* $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot 2\sqrt{2}}{1-(2\sqrt{2})^2} = \frac{4\sqrt{2}}{1-8} = \frac{4\sqrt{2}}{-7} = \boxed{-\frac{4\sqrt{2}}{7}}$

(61) $\sin\theta = \frac{4}{5}$, $\left(\frac{\pi}{2}, \pi\right)$



$*$ $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{4}{5}\cdot\frac{-3}{5} = \boxed{\frac{-24}{25}}$

$*$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{-3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = \boxed{\frac{-7}{25}}$

$*$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot\frac{4}{-3}}{1-\left(\frac{4}{-3}\right)^2} = \frac{\frac{-8}{3}}{1-\frac{16}{9}} = \frac{\frac{-8}{3}}{\frac{-7}{9}} = \frac{-8}{3}\cdot\frac{-9}{7} = \boxed{\frac{24}{7}}$

Half Angle Formulas

(63) $\sin 75^\circ = \sin\left(\frac{150^\circ}{2}\right) = +\sqrt{\frac{1-\cos 150^\circ}{2}} = \sqrt{\frac{1-\frac{-\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

(65) $\tan 67.5^\circ = \tan\left(\frac{135^\circ}{2}\right) = \frac{1-\cos 135^\circ}{\sin 135^\circ} = \frac{1-\frac{-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2+\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2+\sqrt{2}}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{2}} = \frac{2+\sqrt{2}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}+2}{2} = \boxed{\sqrt{2}+1}$

(67) $\sin\frac{15\pi}{8} = \sin\left(\frac{\frac{15\pi}{4}}{2}\right) = \sin\left(\frac{\frac{15\pi}{4}-2\pi}{2}\right)$ coterminal $= \sin\left(\frac{\frac{7\pi}{4}}{2}\right) = -\sqrt{\frac{1-\cos\frac{7\pi}{4}}{2}} = -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$

$= -\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{-\sqrt{2-\sqrt{2}}}{2}}$

$*$ $\frac{15\pi}{8}$ is in the 4^{th} Quad where sin is negative