Acc PreCalc Unit 2 Conics Test Review **Name:** Key

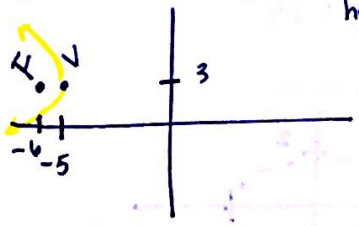
A. Determine the characteristics or equation for each of the following:

ANSWERS

1. What is the focus of the parabola with equation $x = -\frac{1}{4}(y-3)^2 - 5$?

$x+5 = -\frac{1}{4}(y-3)^2$

$-4(x+5) = (y-3)^2$



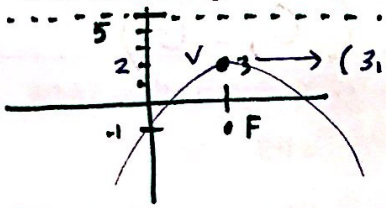
horizontal ↔

$4p = -4$

$p = -1$

1. (-6, 3)

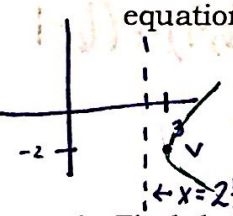
2. Find the equation of the parabola with focus (3, −1) and directrix y = 5.



$p = -3$

2. $(x-3)^2 = -12(y-2)$

3. What is the equation of the directrix of the parabola with equation $x = 4y^2 + 16y + 19$?



$x - 19 + 16 = 4(y^2 + 4y + 4)$

$x - 3 = 4(y+2)^2$

$\frac{1}{4}(x-3) = (y+2)^2$

$V: (3, -2)$

$4p = \frac{1}{4}$

$p = \frac{1}{16}$

$\leftarrow x = 2\frac{15}{16} \leftarrow 3 - \frac{1}{16}$

3. $x = 2\frac{15}{16} = \frac{47}{16} = 2.9375$

4. Find the center of the circle with equation $x^2 + y^2 - 4x + 6y + 1 = 0$.

$x^2 - 4x + 4 + y^2 + 6y + 9 = -1 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 12$

4. (2, -3)

5. Find the radius of the circle with equation $x^2 + y^2 + 2x - 8y - 4 = 0$.

$x^2 + 2x + 1 + y^2 - 8y + 16 = 4 + 1 + 16$

$(x+1)^2 + (y-4)^2 = 21$

5. $\sqrt{21}$

6. Find the equation of a circle with center (1, −4) and radius 5.

6. $(x-1)^2 + (y+4)^2 = 25$

7. Find the center of the ellipse with the equation $3x^2 + 4y^2 + 18x - 32y - 5 = 0$.

$3x^2 + 18x + 4y^2 - 32y = 5$

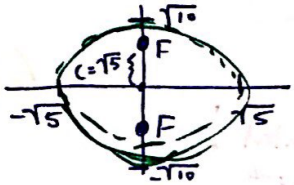
$3(x^2 + 6x + 9) + 4(y^2 - 8y + 16) = 5 + 27 + 64$

$\frac{3(x+3)^2 + 4(y-4)^2 = 96}{96} \rightarrow \frac{(x+3)^2}{32} + \frac{(y-4)^2}{24} = 1$

7. (-3, 4)

8. What are the foci of the ellipse with equation $\frac{2x^2+y^2=10}{10}$?

$\frac{x^2}{5}+\frac{y^2}{10}=1$



$c=\sqrt{a^2-b^2}$

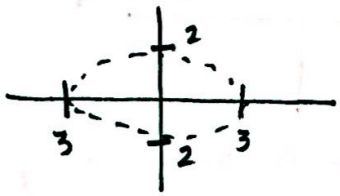
$c=\sqrt{10-5}$

$c=\sqrt{5}$

8. $(0, \pm\sqrt{5})$

9. Find the length of the major axis of the ellipse with equation $\frac{4(x+4)^2+9(y-1)^2=36}{36}$.

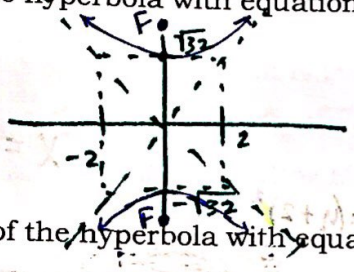
$\frac{(x+4)^2}{9}+\frac{(y-1)^2}{4}=1$



9. 6

10. Find the foci of the hyperbola with equation $\frac{y^2-8x^2=32}{32}$.

$\frac{y^2}{32}-\frac{x^2}{4}=1$



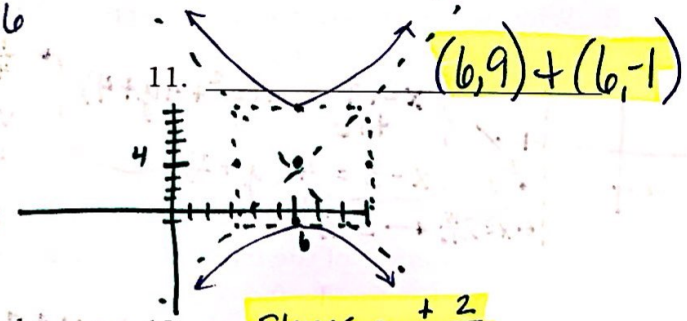
$c=\sqrt{a^2+b^2}$

$c=\sqrt{32+4}$

$c=6$

10. $(0, \pm 6)$

11. Find the vertices of the hyperbola with equation $\frac{(y-4)^2}{25}-\frac{(x-6)^2}{9}=1$.

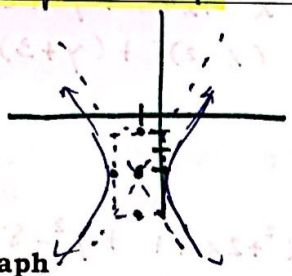


11. $(6,9)+(6,-1)$

12. What are the slopes of the asymptotes of the hyperbola with equation $4x^2-y^2+8x-6y=9$?

$4(x^2+2x+1)-(y^2+6y+9)=9+4-9$

$\frac{4(x+1)^2-(y+3)^2=4}{4} \Rightarrow \frac{(x+1)^2}{1}-\frac{(y+3)^2}{4}=1$



12. Slopes $= \pm\frac{2}{1}$

B. Write the equation in standard form, name the conic section and graph

13. What is the graph of $4x^2=y^2+8y+32$?

$4x^2-y^2-8y=32$

$4x^2-(y^2+8y+16)=32-16$

$\frac{4x^2-(y+4)^2=16}{16}$

$\frac{x^2}{4}-\frac{(y+4)^2}{16}=1$

Asymptotes

$y=\pm\frac{4}{2}(x)-4$

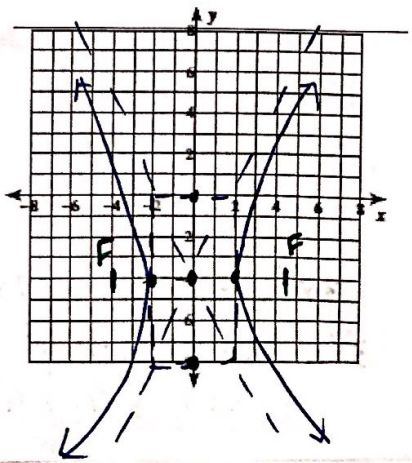
Horizontal Hyperbola

center $(0,-4)$

$c=\sqrt{a^2+b^2}$

$c=\sqrt{16+4}=\sqrt{20}\approx 4.5$

13. Hyperbola



14. What is the graph of $5x^2+10x+5y^2=9$?

$$\frac{5x^2+5y^2+10x=9}{5}$$

$$x^2+y^2+2x=1.8$$

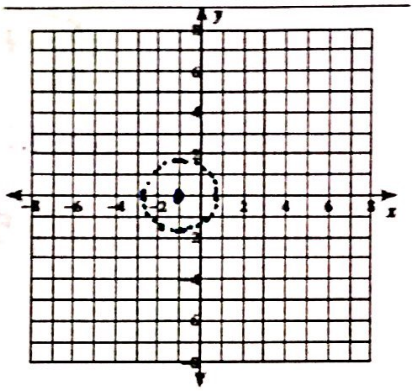
$$(x^2+2x+1)+y^2=1.8+1$$

$$(x+1)^2+y^2=2.8$$

center $(-1,0)$

$r=\sqrt{2.8}\approx 1.67$

14. circle



15. What is the graph of $4x^2=y-24x+35$?

$$4x^2+24x=y+35$$

$$4(x^2+6x+9)=y+35+36$$

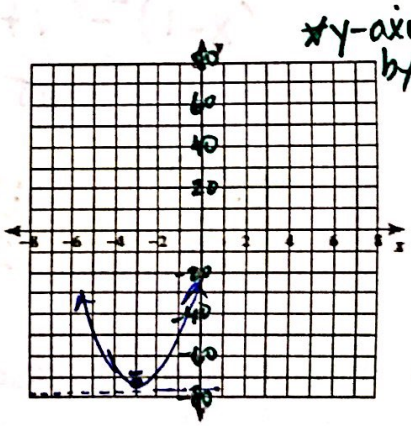
$$4(x+3)^2=y+71$$

$$(x+3)^2=\frac{1}{4}(y+71)$$

vertex: $(-3,-71)$

$4p=\frac{1}{4} \Rightarrow p=\frac{1}{16}$

15. Parabola



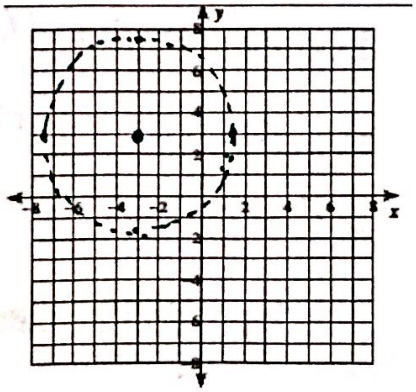
16. What is the graph of: $x^2+y^2+6x-6y=2$?

$$(x^2+6x+\underline{9})+(y^2-6y+\underline{9})=2+9+9$$

$$(x+3)^2+(y-3)^2=20$$

center $(-3,3)$ $r\approx 4.5$

16. circle



17. What is the graph of $x^2 + 25y^2 = 50$?

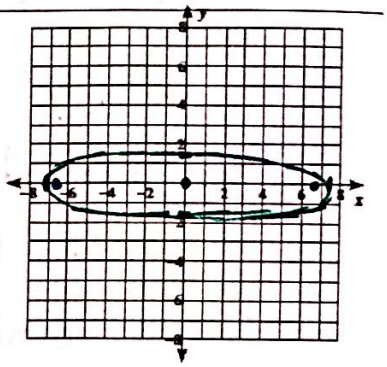
$\frac{x^2 + 25y^2}{50} = \frac{50}{50}$

$\frac{x^2}{50} + \frac{y^2}{2} = 1$

$a \approx 7.1 \quad b \approx 1.4$

$c = \sqrt{50-2} = \sqrt{48} \approx 6.9$

17. Ellipse



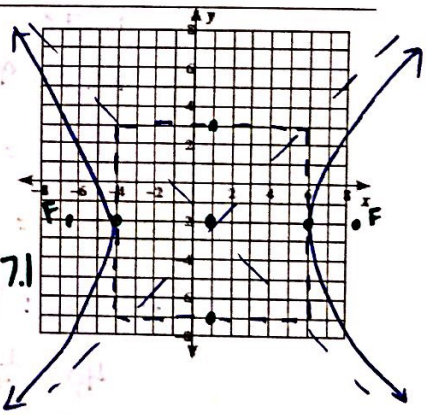
18. What is the graph of $x^2 - y^2 - 2x - 4y = 28$?

$(x^2 - 2x + 1) - (y^2 + 4y + 4) = 28 + 1 + -4$

$\frac{(x-1)^2 - (y+2)^2 = 25}{25}$

$\frac{(x-1)^2}{25} - \frac{(y+2)^2}{25} = 1$

18. Hyperbola



Center $(1,-2)$

$c = \sqrt{a^2+b^2} = \sqrt{25+25} = 7.1$

Asy: $y = \pm\frac{5}{5}(x-1) - 2$

$y = \pm(x-1) - 2$

$y = x - 3$ and $y = -x - 1$

19. Solve the Systems of Equations

$3x^2 + 6y^2 + 13x - y - 67 = 0$

$x - y = -2$

$x = y - 2$

$3(y-2)^2 + 6y^2 + 13(y-2) - y - 67 = 0$

$3(y^2 - 4y + 4) + 6y^2 + 13y - 26 - y - 67 = 0$

$3y^2 - 12y + 12 + 6y^2 + 13y - 26 - y - 67 = 0$

$\frac{9y^2 - 81 = 0}{9} \Rightarrow y^2 - 9 = 0 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$

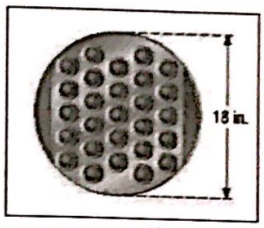
$y = 3 \; ; \; x = 3 - 2 = 1 \Rightarrow (1, 3)$

$y = -3 \; ; \; x = -3 - 2 = -5 \Rightarrow (-5, -3)$

20. Distillation columns are used to separate chemical substances based on the differences in their rates of evaporation. The columns may contain plates with bubble caps or small circular openings. Write an equation for the plate shown, assuming the center is at (-4, 1). What is the surface area of the plate not covered by bubble caps if each cap is 2 inches in diameter?

$(x+4)^2 + (y-1)^2 = 9^2$

$(x+4)^2 + (y-1)^2 = 81$



$r = 9$

$A = \pi r^2$

Area of Plate − Bubble Cap area

$\pi 9^2 - 27(\pi(1)^2)$

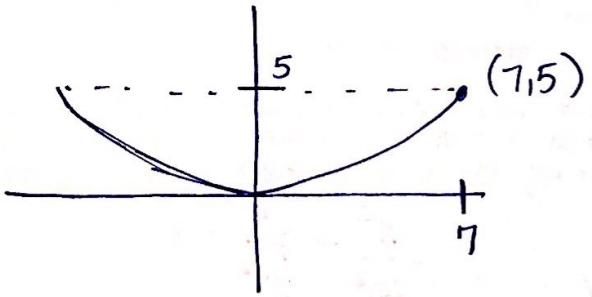
$81\pi - 27\pi$

$54\pi \text{ in}^2$ or 179.1 in^2

21.

A satellite dish is shaped like a paraboloid of revolution. The signals that emanate from a satellite strike the surface of the dish and are reflected to a single point, where the receiver is located. If the dish is 14 feet across at its opening and 5 feet deep at its center, at what position should the receiver be placed?

The receiver should be placed [2.45] feet from the base of the dish, along its axis of symmetry.



$(x-h)^2 = 4p(y-k)$

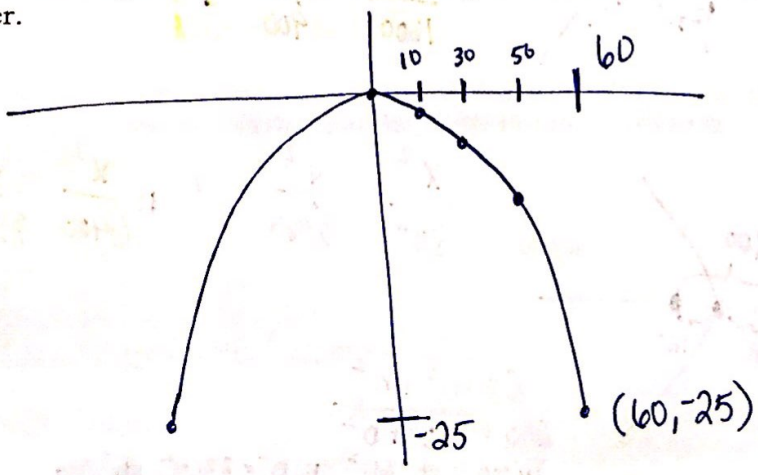
$x^2 = 4py$

$7^2 = 4p(5)$

$49 = 20p$

$2.45 = p$

22. A bridge is built in the shape of a parabolic arch. The bridge has a span of 120 feet and a maximum height of 25 feet. Find the height of the arch at distances of 10, 30, and 50 feet from the center.



$(x-h)^2 = 4p(y-k)$

$x^2 = 4py$

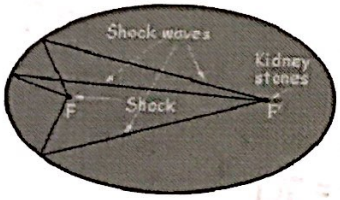
$(60)^2 = 4p(-25)$

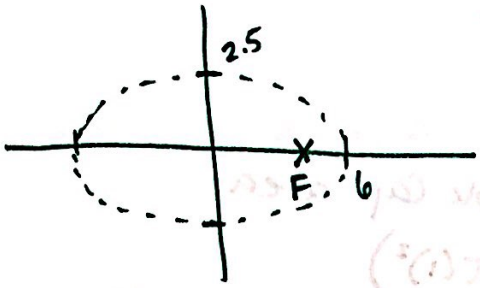
$-36 = p$

$x^2 = -144y$

x	y	height (25−y)
10	$-.69\overline{4}$	$24.3\overline{1}$
30	-6.25	18.75
50	$-17.36\overline{1}$	$7.63\overline{8}$

23. Ellipsoids (3-dimensional ellipse) are used in health care to avoid surgery in the treatment of kidney stones. An elliptical lithotripter emits underwater ultrahigh-frequency (UHF) shock waves from one focus, with the patient's kidney carefully positioned at the other focus. The ellipse used to generate the ellipsoid of a lithotripter has a major axis of 12 ft and a minor axis of 5 ft. How far from the center should the patient's kidney be positioned?





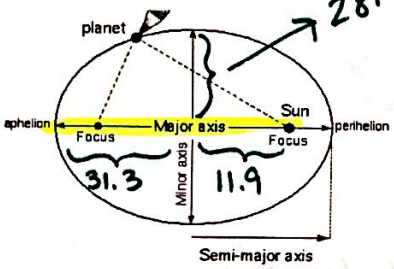
$$\frac{x^2}{6^2} + \frac{y^2}{2.5^2} = 1$$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{6^2 - 2.5^2} \approx 5.45$ ft. from the center

Minor

24. The planets in our solar system orbit the sun on elliptical orbits like the one shown in the diagram. If Mercury's orbit has a semimajor axis of 57.9 Gm (Gigameters) and the Sun (one focus) is 11.9 Gm from the center of the orbit, what is the farthest distance from Mercury to the sun (the aphelion)?



28.95

$c^2 = a^2 - b^2$

$11.9 = \sqrt{a^2 - 28.95^2}$

$141.61 = a^2 - 28.95^2$

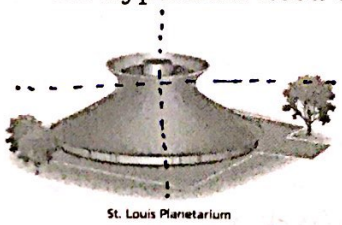
$979.7 \approx a^2$

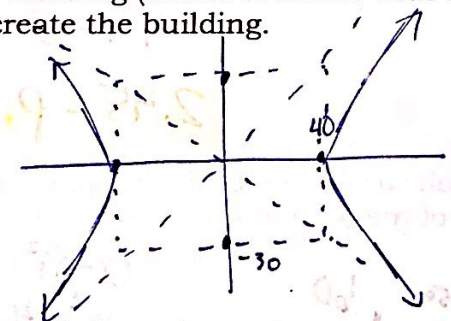
$31.3 \approx a$

Aphellion = 31.3 + 11.9

43.2 Gm

25. The Planetarium in St. Louis, MO is the shape of a hyperboloid (a rotated hyperbola). If the vertices of the hyperbola used to create the hyperbola are 40 ft from the center of the building and 30 ft above the base of the building (which is in line with the central rectangle), write an equation for the hyperbola used to create the building.

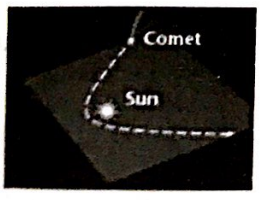

St. Louis Planetarium

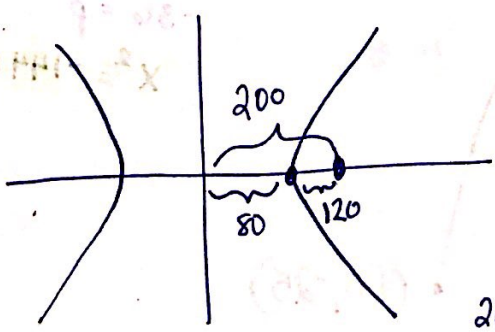


$$\frac{x^2}{40^2} - \frac{y^2}{30^2} = 1$$

$$\frac{x^2}{1600} - \frac{y^2}{900} = 1$$

26. A comet following a hyperbolic path around the sun is 120 Gm (gigameters) from the sun at the vertex. If the Sun (one focus) is 200 Gm from the center of the hyperbola, write an equation to model the hyperbolic path.





$$\frac{x^2}{80^2} - \frac{y^2}{33600} = 1 \Rightarrow \frac{x^2}{6400} - \frac{y^2}{33600} = 1$$

$c = \sqrt{a^2 + b^2}$

$200 = \sqrt{80^2 + b^2}$

$40000 = 80^2 + b^2 \Rightarrow b^2 = 33600$