Name Key

Unit 3C – Test Review

Graphing: Know how to graph one period all 6 trig functions with transformations. Complete the table for the graphed period only.

1. $y = 2\cos\left(4\theta + \frac{7\pi}{6}\right) + 1$

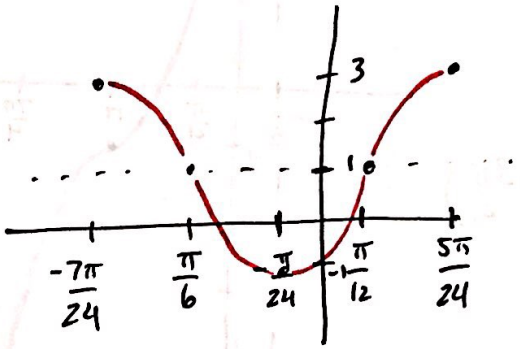
$y = 2\cos\left[4\left(\theta + \frac{7\pi}{24}\right)\right] + 1$

Amp: 2

Per: $\frac{2\pi}{4} = \frac{\pi}{2}$ $\left(\frac{\pi}{2}\cdot\frac{1}{4} = \frac{\pi}{8}\right)$

PS: $\frac{-7\pi}{24}$

VS: 1



x	y
$\frac{-7\pi}{24}$	3
$\frac{-\pi}{6}$	1
$\frac{-\pi}{24}$	-1
$\frac{\pi}{12}$	1
$\frac{5\pi}{24}$	3

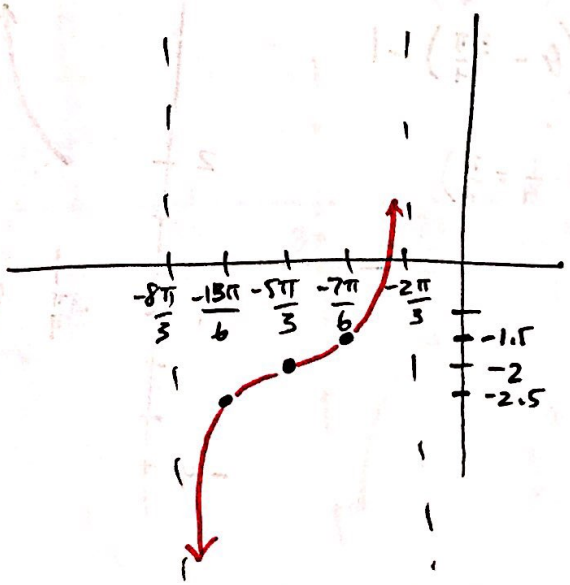
Domain	Range	Asymptote(s)	Max	Min	Odd/Even
$\left[\frac{-7\pi}{24}, \frac{5\pi}{24}\right]$	$[-1, 3]$	none	$y = 3$	$y = -1$	neither

2. $y = \frac{1}{2}\tan\left(\frac{\theta}{2} + \frac{5\pi}{6}\right) - 2$

Asy: $\frac{\theta}{2} + \frac{5\pi}{6} = \frac{-\pi}{2}$ $\theta = \frac{-8\pi}{3}$

and $\frac{\theta}{2} + \frac{5\pi}{6} = \frac{\pi}{2}$ $\theta = \frac{-2\pi}{3}$

Per: $\frac{\pi}{1/2} = 2\pi$ $\left(2\pi\cdot\frac{1}{4} = \frac{\pi}{2}\right)$



x	y
$\frac{-8\pi}{3}$	und
$\frac{-13\pi}{6}$	-2.5
$\frac{-5\pi}{3}$	-2
$\frac{-7\pi}{6}$	-1.5
$\frac{-2\pi}{3}$	und

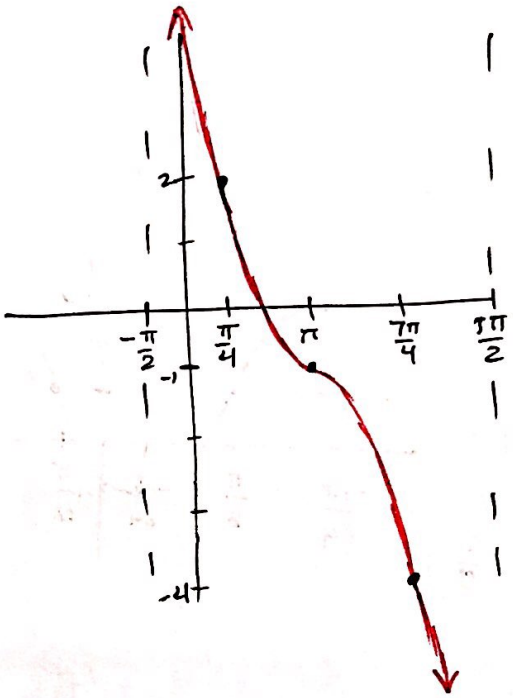
Domain	Range	Asymptote(s)	Max	Min	Odd/Even
$\left(\frac{-8\pi}{3}, \frac{-2\pi}{3}\right)$	$\mathbb{R}$	$x = \frac{-8\pi}{3}$, $x = \frac{-2\pi}{3}$	no	no	neither

3) $y = 3\cot\left(\frac{\theta}{3} + \frac{\pi}{6}\right) - 1$

Asy: $\frac{\theta}{3} + \frac{\pi}{6} = 0 \quad \theta = \frac{-\pi}{2}$

$\frac{\theta}{3} + \frac{\pi}{6} = \pi \quad \theta = \frac{5\pi}{2}$

Per: $\frac{\pi}{1/3} = 3\pi \quad \left(3\pi \cdot \frac{1}{4} = \frac{3\pi}{4}\right)$



x	y
$-\frac{\pi}{2}$	und
$\frac{\pi}{4}$	2
π	-1
$\frac{7\pi}{4}$	-4
$\frac{5\pi}{2}$	und

Domain	Range	Asymptote(s)	Max	Min	Odd/Even
$\left(-\frac{\pi}{2}, \frac{5\pi}{2}\right)$	$\mathbb{R}$	$\theta = \frac{-\pi}{2}$ $\theta = \frac{5\pi}{2}$	no	no	neither

4) $y = -1 + 3\sec\left(\theta - \frac{3\pi}{4}\right)$

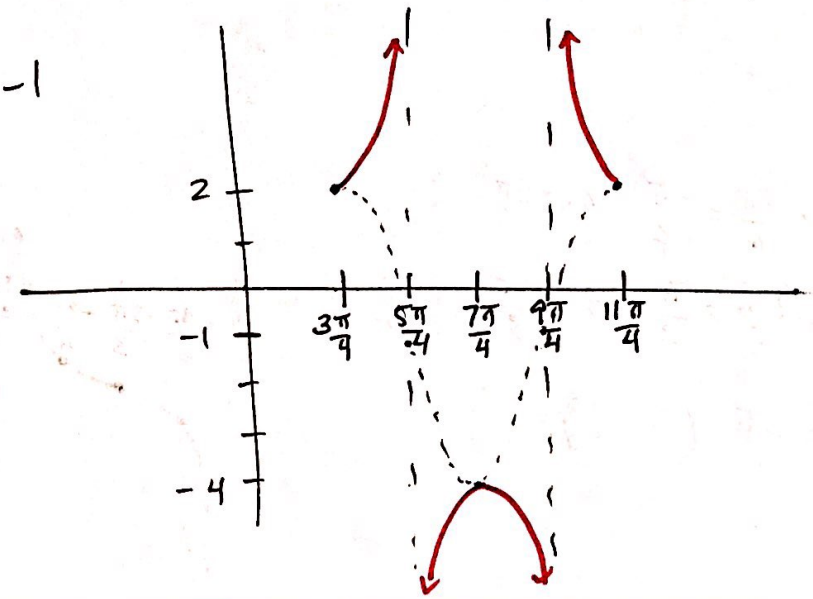
$y = 3\sec\left(\theta - \frac{3\pi}{4}\right) - 1$

Amp: 3

Per: $2\pi \quad \left(2\pi \cdot \frac{1}{4} = \frac{\pi}{2}\right)$

PS: $\frac{3\pi}{4}$

VS: -1



Domain	Range	Asymptote(s)	Max	Min	Odd/Even
$\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{9\pi}{4}\right) \cup \left(\frac{9\pi}{4}, \frac{11\pi}{4}\right]$	$(-\infty, -4] \cup [2, \infty)$	$\theta = \frac{5\pi}{4}$ $\theta = \frac{9\pi}{4}$	no	no	neither

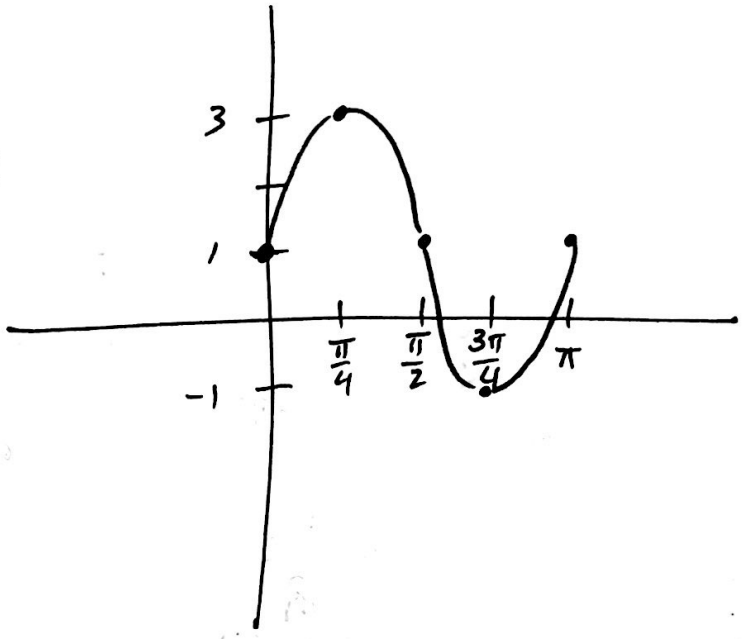
5) $y = 1 + 2\sin 2\theta$

A: 2

Per: $\frac{2\pi}{2} = \pi$ $\left(\pi \cdot \frac{1}{4} = \frac{\pi}{4}\right)$

P.S: none

VS: 1



Domain	Range	Asymptote(s)	Max	Min	Odd/Even
$[0, \pi]$	$[-1, 3]$	no	$y = 3$	$y = -1$	neither

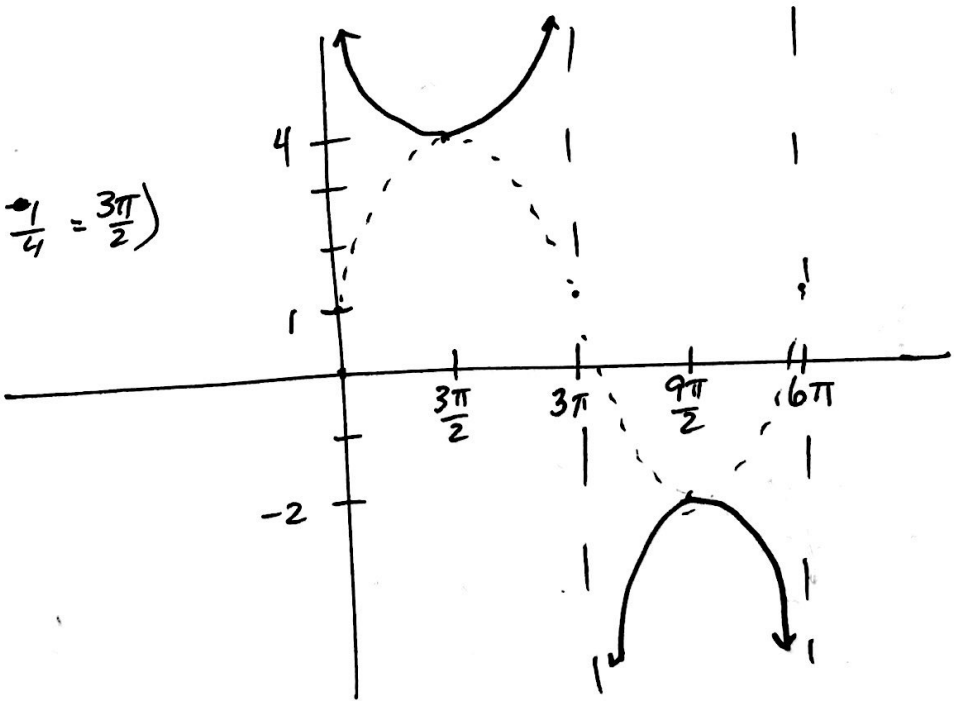
6) $y = 3\csc\left(\frac{\theta}{3}\right) + 1$

A: 3

Per: $\frac{2\pi}{1/3} = 6\pi$ $\left(6\pi \cdot \frac{1}{4} = \frac{3\pi}{2}\right)$

PS: none

VS: 1

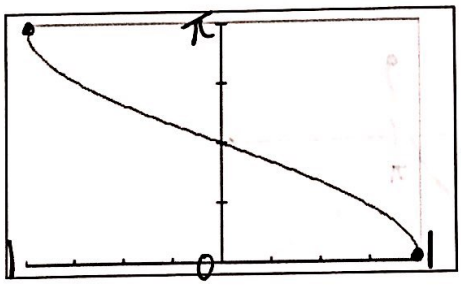


Domain	Range	Asymptote(s)	Max	Min	Odd/Even
$(0, 3\pi) \cup (3\pi, 6\pi)$	$(-\infty, -2] \cup [4, \infty)$	$x = 0$ $x = 3\pi$ $x = 6\pi$	no	no	neither

No calculator

Graphs of Inverses

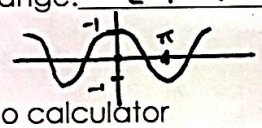
Identify the Graph, State the Domain and Range of Each.

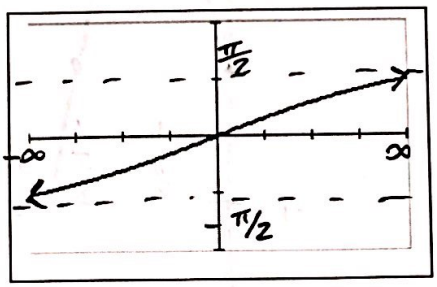


y = $\cos^{-1} x$

Domain: $[-1, 1]$

Range: $[0, \pi]$

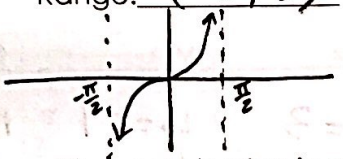


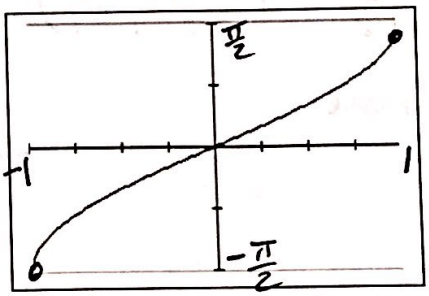


y = $\tan^{-1} x$

Domain: $\mathbb{R}$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

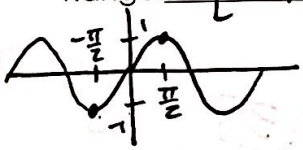




y = $\sin^{-1} x$

Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



No calculator

Inverses: Evaluate the expression. Give exact value in radian (if answer is an angle)

1. Arcsin (–1)

$-\frac{\pi}{2}$

2. Arccos 1

0

3. Arctan (–1)

$-\frac{\pi}{4}$

4. $\text{Cos}^{-1} \frac{1}{2}$

$\frac{\pi}{3}$

5. Arcsin 1

$\frac{\pi}{2}$

6. $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

$-\frac{\pi}{6}$

7. $\cos\left(\text{Cos}^{-1}\left(-\frac{1}{2}\right)\right)$

$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

8. $\sin\left(\text{Sin}^{-1}\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{3}}{2}$

9. $\tan\left(\text{Tan}^{-1}\frac{\sqrt{3}}{3}\right)$

$\frac{\sqrt{3}}{3}$

10. $\text{Cos}^{-1}\left(\text{Cos}\frac{\pi}{2}\right)$

$\frac{\pi}{2}$

11. $\text{Sin}^{-1}\left(\sin\frac{\pi}{4}\right)$

$\frac{\pi}{4}$

12. $\text{Tan}^{-1}\left(\tan\frac{\pi}{3}\right)$

$\frac{\pi}{3}$

13. $\cos\left(\text{Arcsin}\frac{1}{2}\right)$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

14. $\sin\left(\text{Arccos}\frac{\sqrt{3}}{2}\right)$

$\sin\frac{\pi}{6} = \frac{1}{2}$

15. $\tan\left(\text{Arcsin}\frac{\sqrt{3}}{2}\right)$

$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

No calculator

Solve each equation:

1. $\tan x + 12 = 2\tan x + 11$

$-\tan x \quad -\tan x$

$12 = \tan x + 11$

$-11 \quad -11$

$1 = \tan x$

$x = \frac{\pi}{4}$

2. $3\sin x - \sqrt{3} = \sin x$

$-\sin x \quad \sin x$

$2\sin x - \sqrt{3} = 0$

$\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}$

3. $6\cos^2 x + 5\cos x - 4 = 0$ (factor)

$6x^2 + 5x - 4$

$(3x + 4)(2x - 1)$

$8x$

$-3x$

$(3\cos x + 4)(2\cos x - 1) = 0$

$3\cos x + 4 = 0$

$\cos x = \frac{-4}{3}$

↑ no solution

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

3. Changed to

$2\cos^2 x = 1 - \cos x$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

4. $3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\sqrt{\frac{1}{3}}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}$ and $-\frac{\pi}{6}$

5. $\sin^3 x = \sin x$

$-\sin x \quad \sin x$

$\sin^3 x - \sin x = 0$

$\sin x(\sin^2 x - 1) = 0$

$\sin x = 0$

$x = 0$

$\sin^2 x - 1 = 0$

$\sin^2 x = 1$

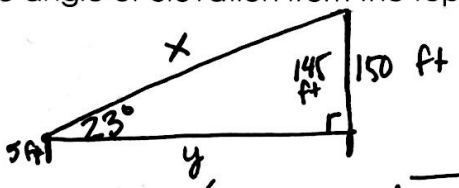
$\sin x = \pm\sqrt{1}$

$\sin x = \pm 1$

$x = \frac{\pi}{2}$ and $-\frac{\pi}{2}$

Applications: Draw diagrams to illustrate the problem. Round to the nearest hundredth.

1) A steel cable zip-line is being constructed for competition on a reality television show. One end of the zip-line is attached to a platform on top of a 150 foot pole. The other end of the zip-line is attached to the top of a 5 foot stake. The angle of elevation from the top of the stake to the top of the platform is 23°.



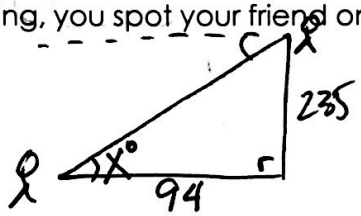
a) How long is the zip-line?

$x \cdot \sin 23° = \frac{145}{x} \cdot x$ $\quad \frac{x \sin 23°}{\sin 23°} = \frac{145}{\sin 23°}$ $\quad \boxed{x = 371.1 \text{ ft}}$

b) How far is the stake from the pole?

$\tan 23° = \frac{145}{y}$ $\quad y = \frac{145}{\tan 23°}$ $\quad \boxed{y = 341.6 \text{ ft}}$

2) Standing on top of a 235 foot tall building, you spot your friend on the ground who is 94 feet away from the building.



a) What is the angle of depression you had to look to spot your friend?

$\tan x° = \frac{235}{94}$ $\quad x° = \tan^{-1}\left(\frac{235}{94}\right)$ $\quad \boxed{x = 68.2°}$

b) What is the distance between you and your friend?

$\cos 68.2° = \frac{94}{x}$ $\quad x = \frac{94}{\cos 68.2°}$ $\quad \boxed{x = 253.1 \text{ ft}}$

3) Regression: The highest recorded temp are recorded each month for Salt Lake City.

Salt Lake City – USA

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar
36.5	43.2	51.8	65.1	72.0	82.9	92.3	90.0	79.3	66.0	49.8	38.3	36.1	42.9	51.0

Use the given data to find:

A. The regression equation: $y = 26.56 \sin(.53x - 2.17) + 64.21$

B. The average temperature:
V.S. $64.21°$

C. The period. Based on the period is the equation a good fit?
$\frac{2\pi}{.53} = 11.9$ Yes b/c we expect the period to be 12 months.

D. What does the amplitude represent?
$26.56°$ represents the variation above and below the average temp of $64.21°$.