Solving Trig Equations

① $\tan\theta + \sqrt{3} = 0 \quad 0 \le \theta < 2\pi$

$\tan\theta = -\sqrt{3}$

$\boxed{\theta = \frac{2\pi}{3}, \frac{5\pi}{3}}$

② $2\sin(\theta + 47°) = 1 \quad 0° \le \theta < 360°$

$\sin(\theta + 47°) = \frac{1}{2}$

$\theta + 47° = 30°, 150°, 390°$

$-47° \quad -47° \quad -47° \quad -47°$

$\boxed{\theta = -17°, 103°, 343°}$

③ $4\cos^2\theta = 1 \quad 0 \le \theta < 2\pi$

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \pm\sqrt{\frac{1}{4}}$

$\cos\theta = \pm\frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

④ $2\sin\theta\cos\theta = \sqrt{2}\cos\theta \quad 0 \le \theta < 2\pi$

$2\sin\theta\cos\theta - \sqrt{2}\cos\theta = 0$

$\cos\theta(2\sin\theta - \sqrt{2}) = 0$

$\cos\theta = 0 \qquad 2\sin\theta - \sqrt{2} = 0$

$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}} \qquad \sin\theta = \frac{\sqrt{2}}{2}$

$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}}$

For all values of θ

$\cos\theta = 0 \qquad \sin\theta = \frac{\sqrt{2}}{2}$

$\boxed{\theta = \frac{\pi}{2} + n\pi} \qquad \boxed{\theta = \frac{\pi}{4} + 2n\pi \quad \theta = \frac{3\pi}{4} + 2n\pi}$

(5) $\tan\theta - \sqrt{3} = 2\tan\theta \quad 0 \le \theta < 2\pi$

$-2\tan\theta \qquad -2\tan\theta$

$-\tan\theta - \sqrt{3} = 0$

$\tan\theta = -\sqrt{3}$

$\boxed{\theta = \frac{2\pi}{3}, \frac{5\pi}{3}}$

For all values of θ

$\tan\theta = -\sqrt{3}$

$\boxed{\theta = \frac{2\pi}{3} + n\pi}$

(6) $2\sin^2\theta + \sin\theta = 0 \quad 0 \le \theta < 2\pi$

$\sin\theta(2\sin\theta + 1) = 0$

$\sin\theta = 0 \qquad 2\sin\theta + 1 = 0$

$\boxed{\theta = 0, \pi} \qquad \sin\theta = -\frac{1}{2}$

$\boxed{\theta = \frac{7\pi}{6}, \frac{11\pi}{6}}$

(7) $2\cos^2\theta - 5\cos\theta + 2 = 0 \quad 0 \le \theta < 2\pi$

$(2\cos\theta - 1)(\cos\theta - 2) = 0$

$2\cos\theta - 1 = 0 \qquad \cos\theta - 2 = 0$

$\cos\theta = \frac{1}{2} \qquad \cos\theta = 2$

no solution

$\boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

(8) $\sin^2\theta + 5\sin\theta + 6 = 0 \quad 0 \le \theta < 2\pi$

$(\sin\theta + 2)(\sin\theta + 3) = 0$

$\sin\theta + 2 = 0 \qquad \sin\theta + 3 = 0$

$\sin\theta = -2 \qquad \sin\theta = -3$

no solution no solution

⑨ $\tan^2\theta - \sec\theta - 1 = 0$ $\quad 0 \le \theta < 2\pi$

$\sec^2\theta - 1 - \sec\theta - 1 = 0$

$\sec^2\theta - \sec\theta - 2 = 0$

$(\sec\theta + 1)(\sec\theta - 2) = 0$

$\sec\theta + 1 = 0 \qquad \sec\theta - 2 = 0$

$\sec\theta = -1 \qquad \sec\theta = 2$

$\frac{1}{\cos\theta} = -1 \qquad \frac{1}{\cos\theta} = 2$

$\cos\theta = -1 \qquad \cos\theta = \frac{1}{2}$

$\theta = \pi \qquad \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

For all values of θ

$\cos\theta = -1$	$\cos\theta = \frac{1}{2}$
$\theta = \pi + 2n\pi$	$\theta = \frac{\pi}{3} + 2n\pi$
	$\theta = \frac{5\pi}{3} + 2n\pi$

⑩ $1 - \cos\theta = -\sin\theta \quad 0 \le \theta < 2\pi$

$(1 - \cos\theta)^2 = (-\sin\theta)^2$

$(1 - \cos\theta)(1 - \cos\theta) = (-\sin\theta)^2$

$1 - 2\cos\theta + \cos^2\theta = \sin^2\theta$

$1 - 2\cos\theta + \cos^2\theta = 1 - \cos^2\theta$

$2\cos^2\theta - 2\cos\theta = 0$

$2\cos\theta(\cos\theta - 1) = 0$

$2\cos\theta = 0 \qquad \cos\theta - 1 = 0$

$\cos\theta = 0 \qquad \cos\theta = 1$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \theta = 0$

check for extraneous solutions b/c we squared both sides:

$1 - \cos\frac{\pi}{2} = -\sin\frac{\pi}{2}$

$1 - 0 = -1$

×

$1 - \cos 0 = -\sin 0$

$1 - 1 = -0$

✓

$1 - \cos\frac{3\pi}{2} = -\sin\frac{3\pi}{2}$

$1 - 0 = -(-1)$

✓

(11) $4\sin\theta\cos\theta = \sqrt{3} \quad 0 \le \theta < 2\pi$

$2 \cdot \underbrace{2\sin\theta\cos\theta}_{\text{double angle}} = \sqrt{3}$

$2 \cdot \sin 2\theta = \sqrt{3}$

$\sin 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{3}+2\pi = \frac{7\pi}{3}, \frac{2\pi}{3}+2\pi = \frac{8\pi}{3}$

$\boxed{\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}}$

(12) $\frac{\sin(90^\circ-\theta)}{\sin\theta} = -\sqrt{3} \quad 0^\circ \le \theta < 360^\circ$

* co-function identity

$\frac{\cos\theta}{\sin\theta} = -\sqrt{3}$

$\cot\theta = -\sqrt{3}$

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\tan\theta = \frac{-\sqrt{3}}{3}$

$\boxed{\theta = 150^\circ, 330^\circ}$

(13) $\sin 2\theta\cos 64^\circ + \cos 2\theta \sin 64^\circ = \frac{\sqrt{3}}{2} \quad 0^\circ \le \theta < 360^\circ$

* sum formula

$\sin(2\theta + 64^\circ) = \frac{\sqrt{3}}{2}$

$2\theta + 64^\circ = 60^\circ, 120^\circ, 420^\circ, 480^\circ, 780^\circ, 840^\circ$

$-64^\circ \quad -64^\circ \quad -64^\circ \quad -64^\circ \quad -64^\circ \quad -64^\circ$

$-4^\circ \quad 56^\circ \quad 356^\circ \quad 416^\circ \quad 716^\circ \quad 776^\circ$

$\div 2 \quad \div 2 \quad \div 2 \quad \div 2 \quad \div 2 \quad \div 2$

$\theta = \cancel{-2^\circ} \quad 28^\circ \quad 178^\circ \quad 208^\circ \quad 358^\circ \quad \cancel{388^\circ}$

(14) $5\sec^2\theta + 2\tan\theta - 8 = 0 \quad 0° \le \theta < 360°$

$5(\tan^2\theta + 1) + 2\tan\theta - 8 = 0$

$5\tan^2\theta + 5 + 2\tan\theta - 8 = 0$

$5\tan^2\theta + 2\tan\theta - 3 = 0$

$(5\tan\theta - 3)(\tan\theta + 1) = 0$

$5\tan\theta - 3 = 0 \qquad \tan\theta + 1 = 0$

$\tan\theta = \frac{3}{5} \qquad \tan\theta = -1$

$\theta \approx 31°, 211°$ (used calculator) $\qquad \theta = 135°, 315°$

(15) $\sin^2\theta + \sin\theta - 1 = 0 \quad 0 \le \theta < 2\pi$

$x^2 + x - 1 = 0$

does not factor... Quad Formula

$\sin\theta = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$

$\sin\theta \approx .618 \qquad \sin\theta \approx -1.618$ no solution

$\theta \approx .67$ rad, 2.48 rad

$\pi - .67$ rad

(16) $\cos 3\theta = \frac{1}{2} \quad 0 \le \theta < 2\pi$

$3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}, \frac{5\pi}{3} + 2\pi = \frac{11\pi}{3}, \frac{7\pi}{3} + 2\pi = \frac{13\pi}{3}, \frac{11\pi}{3} + 2\pi = \frac{17\pi}{3}$

$\frac{3\theta}{3}$ $\div 3$ $\div 3$ $\div 3$ $\div 3$ $\div 3$ $\div 3$

$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$