

Partitioning Line Segments NOTES

Coordinates of point which partitions a line segment AB at the ratio of $a:b$ from $A(x_1, y_1)$ to $B(x_2, y_2)$

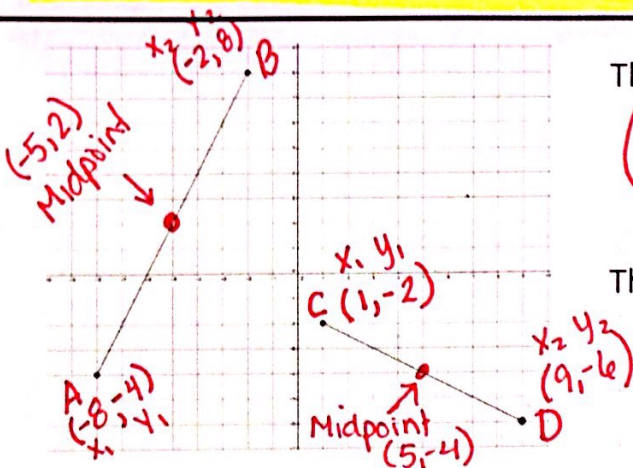
$$(x, y) = \frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}$$

OR

$$(x, y) = \left(x_1 + \frac{a}{a+b} \underbrace{(x_2 - x_1)}_{\text{run}}, y_1 + \frac{a}{a+b} \underbrace{(y_2 - y_1)}_{\text{rise}} \right)$$

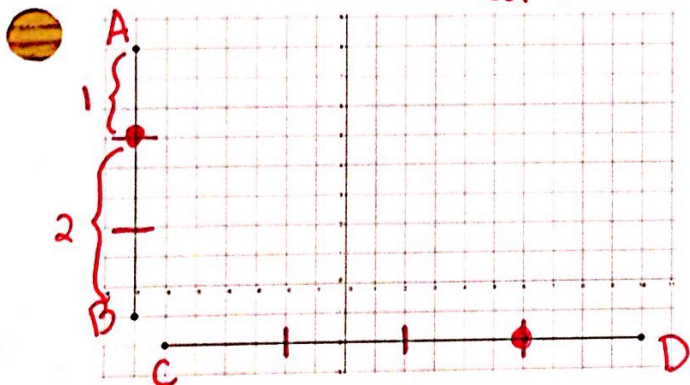
Midpoint Formula = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

The midpoint divides or "partitions" a segment in half, so the lengths have a ratio of 1 : 1.



The midpoint of AB = $\left(\frac{-8 + -2}{2}, \frac{-4 + 8}{2} \right) = (-5, 2)$

The midpoint of CD = $\left(\frac{1 + 9}{2}, \frac{-2 + -6}{2} \right) = (5, -4)$

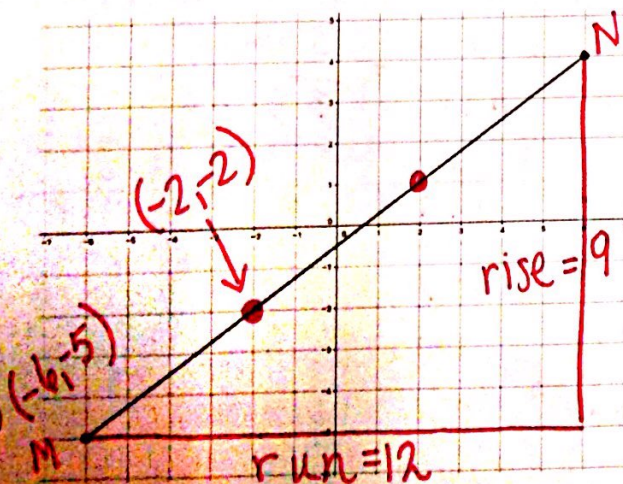


Find the point $\frac{1}{3}$ of the way from A to B.

(-7, 5) partitions AB in a ratio of 1:2.

Find the point $\frac{3}{4}$ of the way from C to D.

(6, -2) partitions $\frac{CD}{AB}$ in a ratio of 3:1.



Find the point $\frac{1}{3}$ of the way from M to N.

*Add $\frac{1}{3}$ of the "run" to the x-value of M

*Add $\frac{1}{3}$ of the "rise" to the y-value of M

$$M = (-6, -5)$$

$$P = \left(-6 + \frac{1}{3}(12), -5 + \frac{1}{3}(9) \right) = (-2, -2)$$

(-2, -2) partitions MN in a ratio of 1:2.

1. Find the midpoint of \overline{AB} , where $A(-12, 18)$ and $B(-6, 24)$.

$$M = \left(\frac{-12 + -6}{2}, \frac{18 + 24}{2} \right)$$

$$M = (-9, 21)$$

2. Find the point P that partitions the line segment \overline{AB} in the given ratio.

$$\begin{array}{ccc} x_1, y_1 & x_2, y_2 & a:b \\ A(-3, 7) & B(5, -9) & \text{Ratio } 1:3 \end{array}$$

$$\left(x_1 + \frac{a}{a+b}(x_2 - x_1), y_1 + \frac{a}{a+b}(y_2 - y_1) \right)$$

$$\left(-3 + \frac{1}{1+3}(5 - -3), 7 + \frac{1}{1+3}(-9 - 7) \right)$$

$$(-1, 3)$$

3. Find the point that is $\frac{2}{3}$ the distance from the endpoint $(-4, -6)$ of the segment with endpoint $(-4, -6)$ and $(2, 9)$.

$$\begin{array}{ccc} x_1, y_1 & x_2, y_2 & a:b \\ (-4, -6) & (2, 9) & 2:1 \end{array}$$

$$\frac{a}{a+b} = \frac{2}{3}$$
$$\frac{2}{2+1}$$

$$\left(x_1 + \frac{a}{a+b}(x_2 - x_1), y_1 + \frac{a}{a+b}(y_2 - y_1) \right)$$

$$\left(-4 + \frac{2}{3}(2 - -4), -6 + \frac{2}{3}(9 - -6) \right)$$

$$(0, 4)$$