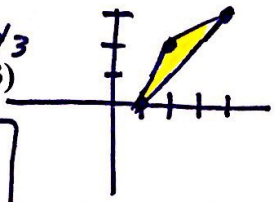
Use Matrices to find Area of a Triangle and Vertex Edge Graphs

****Area of a Triangle**

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Example 1: Find the area of a triangle whose vertices are (1, 0), (2, 2), and (4, 3)

$x_1\ y_1 \quad x_2\ y_2 \quad x_3\ y_3$



$$A = \pm\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & 2 \\ 4 & 3 \end{matrix}$$

$$= \pm\frac{1}{2}\left[(2+0+6) - (8+3+0)\right]$$

$$= \pm\frac{1}{2}[-3] = \frac{3}{2} \text{ or } 1.5 = \text{Area}$$

units^2

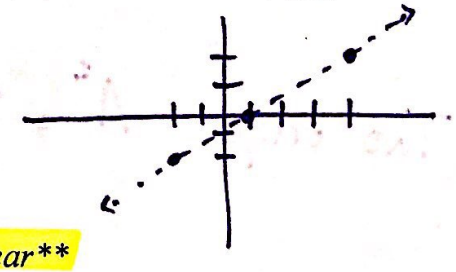
You Try: Find the area of a triangle whose vertices are (1, 2) (6, 2), and (4, 0)

$$A = \pm\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 6 & 2 & 1 \\ 4 & 0 & 1 \end{vmatrix} = 5 \text{ units}^2$$

****Testing for Collinear Points**

Example 2: Determine if the points (-2, -2), (1, 0), and (4, 2) lie of the same line

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} -2 & -2 & 1 \\ 1 & 0 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 0$$



If the area is equal to __0__ then the points are collinear

You Try: Determine if the points (1, 3), (-2, 6), (-1, 1) are collinear

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 6 \text{ units}^2 \Rightarrow \text{The pts. are not collinear.}$$

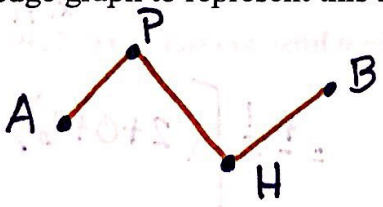
Applications: Path & network can be represented with V.E. graphs
ex: bus routes, computer networks, electrical circuit, food chains

Edge: distance, path, commonality
Vertex: person, place

**Vertex-Edge Graphs

A railway serves four cities, Harrisburg, Baltimore, Philadelphia, and Atlantic City. Trains travel between Harrisburg and Baltimore, Harrisburg and Philadelphia, and Philadelphia and Atlantic City.

A) Draw a vertex edge graph to represent this situation



B) Create the adjacency matrix, A, to represent the vertex-edge graph.

One-edge

$$A^1 = \begin{array}{c} \\ A \\ B \\ H \\ P \end{array} \begin{array}{c} \begin{array}{cccc} A & B & H & P \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

C) Use your calculator to find A^2 and describe what the elements of the new matrix represent.

Two edge

$$A^2 = \begin{array}{c} \\ A \\ B \\ H \\ P \end{array} \begin{array}{c} \begin{array}{cccc} A & B & H & P \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

* A^2 represents the number of ways to go from one vertex to another using 2 edges