

Half Angle Formulas

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} \quad \cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$$

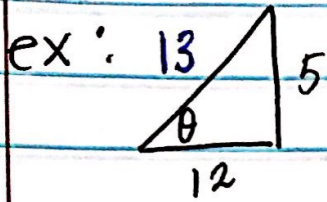
$$\text{ex: } \sin 105^\circ = \sin \left(\frac{210^\circ}{2} \right) = \sqrt{\frac{1 - \cos 210^\circ}{2}} = \sqrt{\frac{1 - \frac{-\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$= \frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$\text{ex: } \tan \frac{11\pi}{12} = \tan \left(\frac{11\pi}{6} \right) = \frac{1 - \cos \frac{11\pi}{6}}{\sin \frac{11\pi}{6}} = \frac{1 - \frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

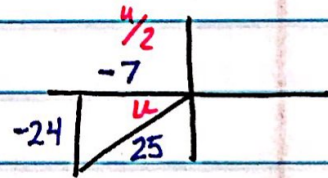
$$= \frac{\frac{2 - \sqrt{3}}{2}}{-\frac{1}{2}} = \frac{2 - \sqrt{3}}{2} \cdot \frac{-2}{1} = -2 + \sqrt{3}$$



$$* \sin \frac{\theta}{2} = + \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \frac{12}{13}}{2}} = \sqrt{\frac{\frac{1}{13}}{2}} = \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26} \cdot \sqrt{26}} = \frac{\sqrt{26}}{26}$$

$$* \sec \frac{\theta}{2} = \sqrt{\frac{2}{1 + \cos \theta}} = \sqrt{\frac{2}{1 + \frac{12}{13}}} = \sqrt{\frac{2}{\frac{25}{13}}} = \sqrt{\frac{2 \cdot 13}{25}} = \frac{\sqrt{26}}{5}$$

ex: $\cos u = -\frac{7}{25}$; $\pi < u < \frac{3\pi}{2}$



$$* \sin \frac{u}{2} = + \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - (-\frac{7}{25})}{2}} = \sqrt{\frac{\frac{32}{25}}{2}} = \sqrt{\frac{32}{25} \cdot \frac{1}{2}} = \sqrt{\frac{32}{50}} = \frac{4}{5}$$

$$* \cos \frac{u}{2} = - \sqrt{\frac{1 + \cos u}{2}} = - \sqrt{\frac{1 + (-\frac{7}{25})}{2}} = - \sqrt{\frac{\frac{18}{25}}{2}} = - \sqrt{\frac{18}{25} \cdot \frac{1}{2}} = - \sqrt{\frac{18}{50}} = - \frac{3}{5}$$

$$* \tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - (-\frac{7}{25})}{\frac{-24}{25}} = \frac{\frac{32}{25}}{\frac{-24}{25}} = \frac{32}{25} \cdot \frac{-25}{24} = -\frac{4}{3}$$

ex: verify the identity

$$\cos^2 2\theta - \sin^2 2\theta = \cos 4\theta$$

$$= \cos(2 \cdot 2\theta)$$

$$= \cos^2 2\theta - \sin^2 2\theta \checkmark$$