"Coded Message String"

23 -31 -45 -9 -7 107 28 -32 -89 19 -38 38

Encryption Decryption Using Matrices

Encoding a Message

If Matrix E is my encryption matrix, to encrypt message Matrix M simply multiply matrix M by matrix E to create the coded message matrix C: **M x E = C**

Given that:

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z	SPACE
14	15	16	17	18	19	20	21	22	23	24	25	26	0

Encoding Example: Use the matrix E to encode the message: Happy Days

Encryption Matrix → $E = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}$

Step 1: Covert Message Characters to Numeric Values

H A P P Y ␣ D A Y S

8 1 16 16 25 0 4 1 25 19

Step 2: Create the Message Matrix and multiply by the Encryption Matrix (encryption matrix will determine the dimensions of the Message Matrix)

$$\underset{4\times 3}{M} \times \underset{3\times 3}{E} = \underset{4\times 3}{C}$$

$$\begin{bmatrix} 8 & 1 & 16 \\ 16 & 25 & 0 \\ 4 & 1 & 25 \\ 19 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix} = \begin{bmatrix} 23 & -31 & -45 \\ -9 & -7 & 107 \\ 28 & -32 & -89 \\ 19 & -38 & 38 \end{bmatrix}$$

Filler numbers

You Try: Use the matrix E to encode the message: Happy Birthday

$E = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$

Happy ␣ Birthday

8 1 16 16 25 0 2 9 8 20 8 4 1 25

$$\begin{bmatrix} 8 & 1 & 16 \\ 16 & 25 & 0 \\ 2 & 9 & 18 \\ 20 & 8 & 4 \\ 1 & 25 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = \begin{bmatrix} -5 & -41 & -87 \\ 91 & 207 & 257 \\ 11 & -5 & -41 \\ 76 & 117 & 227 \\ 40 & 80 & 84 \end{bmatrix}$$

"coded message string" -5 -41 -87 91 207 257 11 -5 -41 76 117 227 40 80 84

Decoding a Message

To Decode matrix C by multiply matrix C by E^{-1} to create message matrix M: $C \times E^{-1} = M$

Example: Decode the cryptograms below

Use the inverse of the matrix $E = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}$ to decode the cryptogram.

21 -40 14 2 -11 32 16 -16 -66 15 -20 -31

Step 1 Multiply the Coded Message Matrix by the inverse of the Encryption Matrix

$$C \cdot E^{-1} = M$$

$$\begin{bmatrix} 21 & -40 & 14 \\ 2 & -11 & 32 \\ 16 & -16 & -66 \\ 15 & -20 & -31 \end{bmatrix} \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}^{-1} = \begin{bmatrix} 19 & 16 & 18 \\ 9 & 14 & 7 \\ 0 & 2 & 18 \\ 5 & 1 & 11 \end{bmatrix}$$

Step 2: Convert the numeric values to characters and write the message

19 16 18 9 14 7 0 2 18 5 1 11

S P R I N G ␣ B R E A K

You Try:

Use the inverse of the matrix $E = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$ to decode the following cryptogram:

a) 20 17 -15 -12 -56 -104 1 -25 -65 62 143 181

$$C \cdot E^{-1} = M$$

$$\begin{bmatrix} 20 & 17 & -15 \\ -12 & -56 & -104 \\ 1 & -25 & -65 \\ 62 & 143 & 181 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}^{-1} = \begin{bmatrix} 19 & 5 & 14 \\ 4 & 0 & 16 \\ 12 & 1 & 14 \\ 5 & 19 & 0 \end{bmatrix}$$

19 5 14 4 0 16 12 1 14 5 19 0

S E N D ␣ P L A N E S ␣