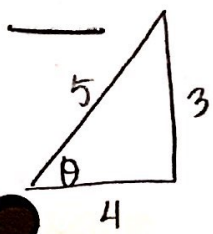


Double & Half Angles



$$\textcircled{1} \sin \theta = \frac{3}{5}$$

$$\textcircled{3} \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

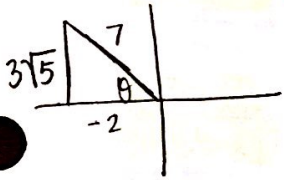
$$\cos 2\theta = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\textcircled{5} \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\tan 2\theta = \frac{2 \cdot \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1 - \frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} \cdot \frac{16}{7} = \frac{24}{7}$$

$$\textcircled{7} \csc 2\theta = \frac{1}{2 \sin \theta \cos \theta} = \frac{1}{2 \left(\frac{3}{5}\right) \left(\frac{4}{5}\right)} = \frac{1}{\frac{24}{25}} = \frac{25}{24}$$

$$\textcircled{22} \cos u = -\frac{2}{7}; \quad \frac{\pi}{2} < u < \pi$$

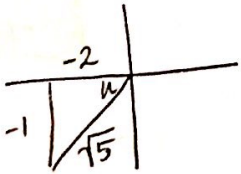


$$\sin 2u = 2 \sin u \cos u = 2 \left(\frac{3\sqrt{5}}{7}\right) \left(-\frac{2}{7}\right) = \frac{-12\sqrt{5}}{49}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{2}{7}\right)^2 - \left(\frac{3\sqrt{5}}{7}\right)^2 = \frac{4}{49} - \frac{45}{49} = \frac{-41}{49}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2 \left(\frac{3\sqrt{5}}{-2}\right)}{1 - \left(\frac{3\sqrt{5}}{-2}\right)^2} = \frac{-3\sqrt{5}}{1 - \frac{45}{4}} = \frac{-3\sqrt{5}}{\frac{4-45}{4}} = -3\sqrt{5} \cdot \frac{4}{-41} = \frac{12\sqrt{5}}{41}$$

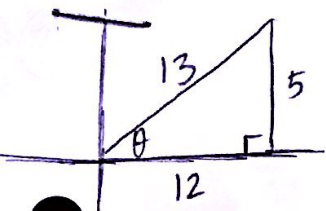
$$\textcircled{23} \tan u = \frac{1}{2}; \quad \pi < u < \frac{3\pi}{2}$$



$$\sin 2u = 2 \sin u \cos u = 2 \left(\frac{-1}{\sqrt{5}}\right) \left(\frac{-2}{\sqrt{5}}\right) = \frac{4}{5}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \left(\frac{-2}{\sqrt{5}}\right)^2 - \left(\frac{-1}{\sqrt{5}}\right)^2 = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2 \left(\frac{-1}{2}\right)}{1 - \left(\frac{-1}{2}\right)^2} = \frac{1}{1 - \frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$



$$(31) \cos \frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\frac{12}{13}}{2}} = \sqrt{\frac{\frac{1}{2} + \frac{12}{26}}{2}} = \sqrt{\frac{13+12}{26}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$$

$$(33) \tan \frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\frac{12}{13}}{1+\frac{12}{13}}} = \sqrt{\frac{\frac{1}{13}}{\frac{25}{13}}} = \sqrt{\frac{1}{13} \cdot \frac{13}{25}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$$

$$(35) \csc \frac{\theta}{2} = \sqrt{\frac{2}{1-\cos\theta}} = \sqrt{\frac{2}{1-\frac{12}{13}}} = \sqrt{\frac{2}{\frac{1}{13}}} = \sqrt{26}$$

$$(39) 15^\circ \quad \text{Quad I} \quad \sin 15^\circ = \sin\left(\frac{30^\circ}{2}\right) = \sqrt{\frac{1-\cos 30^\circ}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{4}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\cos 15^\circ = \cos\left(\frac{30^\circ}{2}\right) = \sqrt{\frac{1+\cos 30^\circ}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{4}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\tan 15^\circ = \tan\left(\frac{30^\circ}{2}\right) = \frac{1-\cos 30^\circ}{\sin 30^\circ} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$

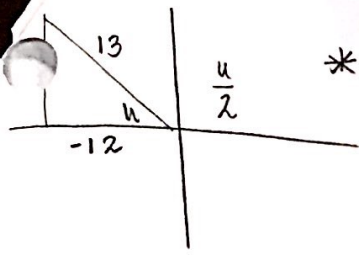
$$(46) \frac{7\pi}{12} \quad \text{Quad II} \quad \sin \frac{7\pi}{12} = \sin\left(\frac{7\pi/6}{2}\right) = \sqrt{\frac{1-\cos \frac{7\pi}{6}}{2}} = \sqrt{\frac{1-\frac{-\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{1+\sqrt{3}}{2}}{4}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\cos \frac{7\pi}{12} = \cos\left(\frac{7\pi/6}{2}\right) = -\sqrt{\frac{1+\cos \frac{7\pi}{6}}{2}} = -\sqrt{\frac{1+\frac{-\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{1-\sqrt{3}}{2}}{4}} = -\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{-\sqrt{2-\sqrt{3}}}{2}$$

$$* \frac{7\pi}{12} \cdot 2 = \frac{7\pi}{6}$$

$$\tan \frac{7\pi}{12} = \tan\left(\frac{7\pi/6}{2}\right) = \frac{1-\cos \frac{7\pi}{6}}{\sin \frac{7\pi}{6}} = \frac{1-\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = \frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2+\sqrt{3}}{1} = 2+\sqrt{3}$$

$$\sin u = \frac{5}{13} ; \frac{\pi}{2} < u < \pi$$

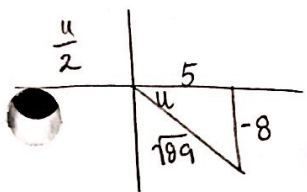


$$\begin{aligned} * \sin \frac{u}{2} &= + \sqrt{\frac{1 - \cos u}{2}} = + \sqrt{\frac{1 - \frac{-12}{13}}{2}} = + \sqrt{\frac{\frac{25}{13}}{2}} = + \sqrt{\frac{25}{13} \cdot \frac{1}{2}} = + \sqrt{\frac{25}{26}} = \frac{+ \sqrt{25}}{\sqrt{26}} \\ &= \frac{+ 5}{\sqrt{26}} = \frac{5\sqrt{26}}{26} \end{aligned}$$

$$* \cos \frac{u}{2} = + \sqrt{\frac{1 + \cos u}{2}} = + \sqrt{\frac{1 + \frac{-12}{13}}{2}} = + \sqrt{\frac{\frac{1}{13}}{2}} = + \sqrt{\frac{1}{13} \cdot \frac{1}{2}} = + \sqrt{\frac{1}{26}} = \frac{+ \sqrt{1}}{\sqrt{26}} = \frac{+ 1}{\sqrt{26}} = \frac{+ \sqrt{26}}{26}$$

$$* \tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{-12}{13}}{\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{5}{13}} = \frac{25}{13} \cdot \frac{13}{5} = \frac{25}{5} = 5$$

$$(49) \tan u = -\frac{8}{5} ; \frac{3\pi}{2} < u < 2\pi$$



$$* \sin \frac{u}{2} = + \sqrt{\frac{1 - \cos u}{2}} = + \sqrt{\frac{1 - \frac{5}{109}}{2}} = + \sqrt{\frac{1 - \frac{5\sqrt{89}}{89}}{2}} = + \sqrt{\frac{89 - 5\sqrt{89}}{89}} = + \sqrt{\frac{89 - 5\sqrt{89}}{178}}$$

$$\sin u = \frac{-8}{109} = -\frac{8\sqrt{89}}{89}$$

$$\cos u = \frac{5}{109} = \frac{5\sqrt{89}}{89}$$

$$= \frac{+ \sqrt{89 - 5\sqrt{89}}}{178}$$

$$* \cos \frac{u}{2} = - \sqrt{\frac{1 + \cos u}{2}} = - \sqrt{\frac{1 + \frac{5\sqrt{89}}{89}}{2}} = - \sqrt{\frac{89 + 5\sqrt{89}}{89}} = - \sqrt{\frac{89 + 5\sqrt{89}}{178}}$$

$$* \tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{5\sqrt{89}}{89}}{-\frac{8\sqrt{89}}{89}} = \frac{89 - 5\sqrt{89}}{89} \cdot \frac{89}{-8\sqrt{89}} = \frac{89 - 5\sqrt{89}}{-8\sqrt{89}}$$

$$= \frac{89 - 5\sqrt{89}}{-8\sqrt{89}} = \frac{(89 - 5\sqrt{89}) \cdot \sqrt{89}}{-8\sqrt{89} \cdot \sqrt{89}} = \frac{89\sqrt{89} - 445}{-712} = \frac{\sqrt{89} - 5}{-8} = \frac{-5 + \sqrt{89}}{-8} = \frac{5 - \sqrt{89}}{8}$$

$$\textcircled{81} \quad \csc 2\theta = \frac{\csc \theta}{2 \cos \theta}$$

$$\csc 2\theta \rightarrow \frac{1}{2 \sin \theta \cos \theta} \rightarrow \frac{\csc \theta}{2 \cos \theta} \quad \checkmark$$

$$\textcircled{84} \quad \cos^4 x - \sin^4 x = \cos 2x$$

$$\underbrace{(\cos^2 x + \sin^2 x)}_1 (\cos^2 x - \sin^2 x) \rightarrow \underbrace{\cos^2 x - \sin^2 x}_{\cos 2x} \rightarrow \cos 2x \quad \checkmark$$

$$\textcircled{85} \quad (\sin x + \cos x)^2 = 1 + \sin 2x$$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x \rightarrow 1 + \underbrace{2 \sin x \cos x}_{\sin 2x} \rightarrow 1 + \sin 2x$$