Double and Half Angles: Complete 1-7 odd, 22, 23, 31, 33,
$35,39,46,47,49,81,84,85$
In Exercises 1-8, use the figure to find the exact value of the trigonometric function.


1. $\sin \theta$
2. $\tan \theta$
3. $\cos 2 \theta$
4. $\sin 2 \theta$
5. $\tan 2 \theta$
6. $\sec 2 \theta$
7. $\csc 2 \theta$
8. $\cot 2 \theta$

In Exercises 21-24, find the exact values of $\sin 2 u$, $\cos 2 u$, and $\tan 2 u$ using the double-angle formulas.
21. $\sin u=\frac{3}{5}, \quad 0<u<\pi / 2$
22. $\cos u=-\frac{2}{7}, \quad \pi / 2<u<\pi$
23. $\tan u=\frac{1}{2}, \quad \pi<u<3 \pi / 2$
24. $\cot u=-6, \quad 3 \pi / 2<u<2 \pi$

In Exercises 31-38, use the figure to find the exact value of the trigonometric function.

31. $\cos \frac{\theta}{2}$
32. $\sin \frac{\theta}{2}$
33. $\tan \frac{\theta}{2}$
34. $\sec \frac{\theta}{2}$
35. $\csc \frac{\theta}{2}$
36. $\cot \frac{\theta}{2}$
37. $2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$
38. $2 \cos \frac{\theta}{2} \tan \frac{\theta}{2}$

In Exercises 39-46, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.
39. $15^{\circ}$
40. $165^{\circ}$
41. $112^{\circ} 30^{\prime}$
42. $157^{\circ} 30^{\prime}$
43. $\frac{\pi}{8}$
44. $\frac{\pi}{12}$
45. $\frac{3 \pi}{8}$
46. $\frac{7 \pi}{12}$

In Exercises 47-50, find the exact values of $\sin (u / 2)$, $\cos (u / 2)$ and $\tan (u / 2)$ using the half-angle formulas.
47. $\sin u=\frac{5}{13}, \quad \pi / 2<u<\pi$
48. $\cos u=\frac{7}{25}, \quad 0<u<\pi / 2$
49. $\tan u=-\frac{8}{5}, \quad 3 \pi / 2<u<2 \pi$
50. $\cot u=7, \quad \pi<u<3 \pi / 2$

In Exercises 81-92, verify the identity algebraically. Use a graphing utility to confirm the identity graphically.
81. $\csc 2 \theta=\frac{\csc \theta}{2 \cos \theta} \quad$ 82. $\sec 2 \theta=\frac{\sec ^{2} \theta}{2-\sec ^{2} \theta}$
83. $\cos ^{2} 2 \alpha-\sin ^{2} 2 \alpha=\cos 4 \alpha$
84. $\cos ^{4} x-\sin ^{4} x=\cos 2 x$
85. $(\sin x+\cos x)^{2}=1+\sin 2 x$

