Sum & Difference Formulas

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

ex: Find the exact value of sin, cos, tan of 165°

$\sin(165°) = \sin(\underset{x}{135°} + \underset{y}{30°})$

$= \sin 135° \cos 30° + \cos 135° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{-\sqrt{2}}{4}$

$\sin(165°) = \dfrac{\sqrt{6} - \sqrt{2}}{4}$

$\cos(165°) = \cos(135° + 30°)$

$= \cos 135° \cos 30° - \sin 135° \sin 30°$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\cos(165°) = \dfrac{-\sqrt{6} - \sqrt{2}}{4}$

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$$\tan(165°) = \tan(135° + 30°)$$

$$= \frac{\tan 135 + \tan 30°}{1 - \tan 135° \cdot \tan 30°}$$

$$= \frac{-1 + \frac{\sqrt{3}}{3}}{1 - (-1 \cdot \frac{\sqrt{3}}{3})} = \frac{-1 + \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$$

$$= \frac{-\frac{3}{3} + \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}} = \frac{\frac{-3+\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{-3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3+\sqrt{3}}$$

$$= \frac{(-3+\sqrt{3})}{(3+\sqrt{3})} \cdot \frac{(3-\sqrt{3})}{(3-\sqrt{3})} = \frac{-9 + 3\sqrt{3} + 3\sqrt{3} - 3}{9 - \cancel{3\sqrt{3}} + \cancel{3\sqrt{3}} - 3}$$

$$= \frac{-12 + 6\sqrt{3}}{6}$$

$$\tan(165°) = -2 + \sqrt{3}$$

ex: Write as a single trig expression, then find the exact value of.

ex: $\cos 25° \cos 20° - \sin 25° \sin 20°$

$$\cos(x+y) = \cos(25° + 20°)$$

$$= \cos 45° = \frac{\sqrt{2}}{2}$$

ex: $\dfrac{\tan\frac{23\pi}{18} - \tan\frac{5\pi}{18}}{1+\tan\frac{23\pi}{18}\tan\frac{5\pi}{18}}$

$\tan(x-y) = \tan\left(\frac{23\pi}{18} - \frac{5\pi}{18}\right)$

$= \tan\left(\frac{18\pi}{18}\right) = \tan\pi = 0$

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ex: $\sin u = \frac{5}{13}$ where $0 < u < \frac{\pi}{2}$

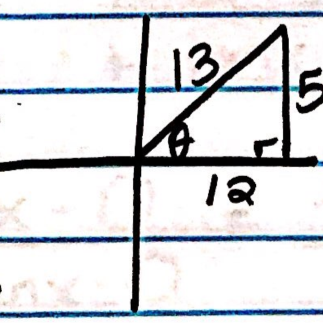
$\cos v = \frac{3}{5}$ where $\frac{3\pi}{2} < v < 2\pi$

Find $\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\frac{12}{13} \cdot \frac{3}{5} - \frac{5}{13} \cdot \frac{-4}{5} = \boxed{\frac{56}{65}}$

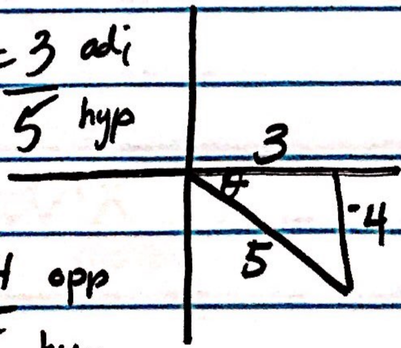
$\sin u = \frac{5 \text{ opp}}{13 \text{ hyp}}$

$\cos u = \frac{12 \text{ adj}}{13 \text{ hyp}}$



$\cos v = \frac{3 \text{ adj}}{5 \text{ hyp}}$

$\sin v = \frac{-4 \text{ opp}}{5 \text{ hyp}}$



Verify the Identity

ex: $\sin(\underbrace{\theta}_{x} + \underbrace{\tfrac{3\pi}{2}}_{y}) = -\cos\theta$

Apply: $\sin(x+y) = \sin x\cos y + \cos x\sin y$

$$\sin\left(\theta + \frac{3\pi}{2}\right) = \sin\theta\cos\frac{3\pi}{2} + \cos\theta\sin\frac{3\pi}{2}$$

$$= \sin\theta \cdot 0 + \cos\theta \cdot {}^{-}1$$

$$= 0 + {}^{-}\cos\theta$$

$$= -\cos\theta$$

ex: $\tan(\underbrace{x}_{x} - \underbrace{180^\circ}_{y}) = \tan x$

Apply: $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x\tan y}$

$$\tan(x - 180^\circ) = \frac{\tan x - \tan 180^\circ}{1 + \tan x \cdot \tan 180^\circ}$$

$$= \frac{\tan x - 0}{1 + \tan x \cdot 0} = \frac{\tan x}{1} = \tan x$$