AccPreCalc Sum and Difference Practice

Name Key

Assigned Problems: #3, 11 (cos only), 13 (sin only), 15 (tan only), 19, 20, 23, 35, 36, 39, 43, 46

In Exercises 1–8, find the exact value of each expression.

3. (a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$ (b) $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3}$

In Exercises 9–18, use the sum and difference formulas to find the exact values of the sine, cosine, and tangent of the angle.

cos 11. $105° = 60° + 45°$

sin 13. $195° = 225° - 30°$

tan 15. $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$

In Exercises 19–28, use the sum and difference formulas to write the expression as the sine, cosine, or tangent of an angle.

19. $\cos 40° \cos 15° - \sin 40° \sin 15°$

20. $\sin 110° \cos 80° + \cos 110° \sin 80°$

21. $\sin 340° \cos 50° - \cos 340° \sin 50°$

22. $\cos 20° \cos 30° + \sin 20° \sin 30°$

23. $\frac{\tan 325° - \tan 86°}{1 + \tan 325° \tan 86°}$ 24. $\frac{\tan 140° - \tan 60°}{1 + \tan 140° \tan 60°}$

In Exercises 35–38, find the exact value of the trigonometric function given that

$\sin u = 5/13$, where $0 < u < \pi/2$

$\cos v = -3/5$, where $\pi/2 < v < \pi$.

35. $\sin(u + v)$ 36. $\cos(v - u)$

37. $\cos(u + v)$ 38. $\sin(u - v)$

In Exercises 39–42, find the exact value of the trigonometric function given that

$\sin u = 7/25$, where $\pi/2 < u < \pi$

$\cos v = 4/5$, where $3\pi/2 < v < 2\pi$.

39. $\cos(u + v)$ 40. $\sin(u + v)$

In Exercises 43–50, verify the identity.

43. $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = 0$

46. $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$

③ a. $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right) = \sin\frac{7\pi}{6}\cos\frac{\pi}{3} - \cos\frac{7\pi}{6}\sin\frac{\pi}{3}$

$= -\frac{1}{2} \cdot \frac{1}{2} - -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$

$= -\frac{1}{4} + \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

b. $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{-1-\sqrt{3}}{2}$

⑪ $\cos 105° = \cos(60° + 45°) = \cos 60° \cos 45° - \sin 60° \sin 45°$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

⑬ $\sin 195° = \sin(225° - 30°) = \sin 225 \cos 30° - \cos 225 \sin 30°$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - -\frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{-\sqrt{6} + \sqrt{2}}{4}$

⑮ $\tan\frac{11\pi}{12} = \tan\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) = \frac{\tan\frac{3\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{3\pi}{4}\tan\frac{\pi}{6}}$

$= \frac{-1 + \frac{\sqrt{3}}{3}}{1 - \left(-1 \cdot \frac{\sqrt{3}}{3}\right)} = \frac{-1 + \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} = \frac{-\frac{3}{3} + \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}} = \frac{\frac{-3+\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{-3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3+\sqrt{3}}$

$= \frac{-3+\sqrt{3}}{3+\sqrt{3}} \cdot \frac{(3-\sqrt{3})}{(3-\sqrt{3})} = \frac{-9 + 3\sqrt{3} + 3\sqrt{3} - 3}{9 - \cancel{3\sqrt{3}} + \cancel{3\sqrt{3}} - 3} = \frac{-12 + 6\sqrt{3}}{6} = -2 + \sqrt{3}$

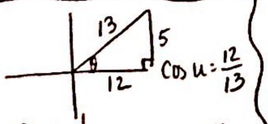
⑲ $\cos 40° \cos 15° - \sin 40° \sin 15° = \cos(40° + 15°) = \cos(55°)$

⑳ $\sin 110° \cos 80° + \cos 110° \sin 80° = \sin(110° + 80°) = \sin(190°)$

㉓ $\frac{\tan 325° - \tan 86°}{1 + \tan 325° \tan 86°} = \tan(325° - 86°) = \tan(239°)$

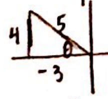
For 35 + 36

$\sin u = \frac{5}{13}, 0 < u < \frac{\pi}{2}$,



$\cos u = \frac{12}{13}$

$\cos v = \frac{-3}{5}, \frac{\pi}{2} < v < \pi$



$\sin v = \frac{4}{5}$

㉟ $\sin(u+v) = \sin u \cos v + \cos u \sin v$

$= \frac{5}{13} \cdot \frac{-3}{5} + \frac{12}{13} \cdot \frac{4}{5}$

$= \frac{-15}{65} + \frac{48}{65} = \frac{33}{65}$

㊱ $\cos(v-u) = \cos v \cos u + \sin v \sin u$

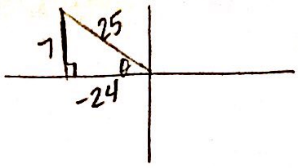
$= \frac{-3}{5} \cdot \frac{12}{13} + \frac{4}{5} \cdot \frac{5}{13}$

$= \frac{-36}{65} + \frac{20}{65} = \frac{-16}{65}$

(39) $\cos(u+v) = \cos u \cos v - \sin u \sin v$

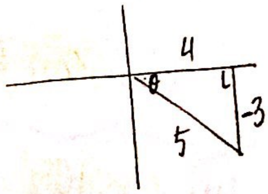
$$\frac{-24}{25} \cdot \frac{4}{5} - \frac{7}{25} \cdot \frac{-3}{5} = \frac{-96}{125} + \frac{21}{125} = \frac{-75}{125} = \frac{-3}{5}$$

$\sin u = \frac{7}{25}$ $\quad \frac{\pi}{2} < u < \pi$



$\cos u = \frac{-24}{25}$

$\cos v = \frac{4}{5}$ $\quad \frac{3\pi}{2} < v < 2\pi$



$\sin v = \frac{-3}{5}$

(43) $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = 0$

$\cos\pi \cos\theta + \sin\pi \sin\theta + \sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta = 0$

$-1 \cdot \cos\theta + 0 \cdot \sin\theta + 1 \cdot \cos\theta + 0 \cdot \sin\theta = 0$

$-\cos\theta + 0 + \cos\theta + 0 = 0$

$0 = 0$

(46) $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$

$$\frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4} \cdot \tan\theta} = \frac{1 - \tan\theta}{1 + 1 \cdot \tan\theta} = \frac{1 - \tan\theta}{1 + \tan\theta}$$