Name Key

Using Coordinates to Prove Theorems about Circles PRACTICE

1. **Use circle O to prove that all radii of a circle have the same length.**

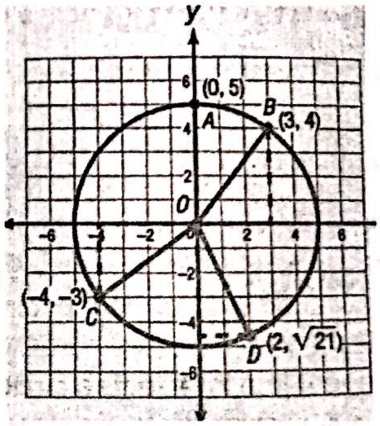
   The length of radius OA is 5 .(Just count)

   Find the lengths of other radii. (Use the distance formula!)

   OB = $\sqrt{(3-0)^2+(4-0)^2} = 5$

   OC = $\sqrt{(-4-0)^2+(-3-0)^2} = 5$

   OD = $\sqrt{(2-0)^2+(\sqrt{21}-0)^2} = 5$



2. **Prove or disprove that the point (13, 16) lies on circle O with center (7, 8) and diameter 20.**

   $r = 10$

   $(x-7)^2+(y-8)^2 = 100$

   $(13-7)^2+(16-8)^2 \stackrel{?}{=} 100$

   $100 \stackrel{\checkmark}{=} 100$

   yes!

3. **The line $y=\frac{1}{4}x+3$ is tangent to the circle $(x-5)^2+y^2=17$ at the point (4, 4). Prove the tangent line is perpendicular to the radius at the point of tangency.**

   center (5,0) pt (4,4)

   Slope of radius $= \frac{4-0}{4-5} = \frac{4}{-1}$

   Slope of the tangent line = $\frac{1}{4}$

   Slope of the radius = $-\frac{4}{1}$

   Answer: The tangent line (is)/is not perpendicular to the radius at the point of tangency because the slopes of the tangent line and of the radius (are)/are not opposite reciprocals.

4. **The line $y=-\frac{2}{3}x+4$ is tangent to the circle $(x+5)^2+(y-3)^2=13$ at the point (-3, 6). Prove the tangent line is perpendicular to the radius at the point of tangency.**

   center (-5,3) pt (-3,6)

   Slope of radius $= \frac{6-3}{-3-(-5)} = \frac{3}{2}$

   Slope of the tangent line = $-\frac{2}{3}$

   Slope of the radius = $\frac{3}{2}$

   Answer: The tangent line (is)/is not perpendicular to the radius at the point of tangency because the slopes of the tangent line and of the radius (are)/are not opposite reciprocals.

5. **Show that the equation of the circle with endpoints on the diameter (4, -1) and (-6, 7) is** $x^2+y^2+2x-6y-31=0$

Center = $(-1, 3)$ (Midpoint formula) Radius = $\sqrt{41}$ (Distance formula)

$\left(\frac{4+-6}{2}, \frac{-1+7}{2}\right)$

$r=\sqrt{(-1-4)^2+(3--1)^2}=\sqrt{41}$

Equation of circle: $(x+1)^2+(y-3)^2=41$

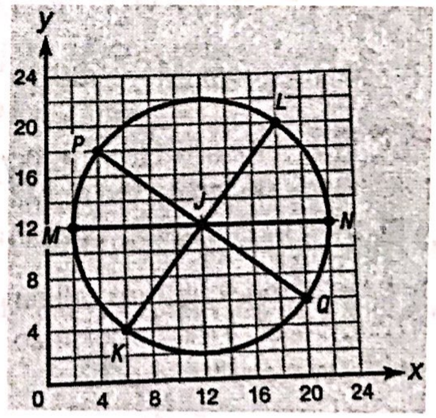
Convert the above equation to general form.

$(x+1)(x+1)+(y-3)(y-3)=41$

$x^2+2x+1+y^2-6y+9=41$

$x^2+y^2+2x-6y-31=0$

6. **Prove that the diameter of circle J is twice the length of its radius.**



diameter MN = 20 (Just count)

radius MJ = 11 (Just count)

diameter KL = $\sqrt{(18-6)^2+(20-4)^2}=20$ (distance formula)

radius KJ = $\sqrt{(12-6)^2+(12-4)^2}=10$ (distance formula)

So, each diameter is half the length of each radius.

7. **Prove or disprove that the point (-9, 3) lies on circle O with center (-5, 3) and contains the point (-1, 4).**

$(x+5)^2+(y-3)^2=17$

$r=\sqrt{(-1--5)^2+(4-3)^2}$

$r=\sqrt{17}$

$(-9+5)^2+(3-3)^2\stackrel{?}{=}17$

$16\neq17$ no!

For 8-9: Graph using the attached graph paper.

8. **Write the equation of the circle with center (10, -14) and tangent to the line x = 15.**

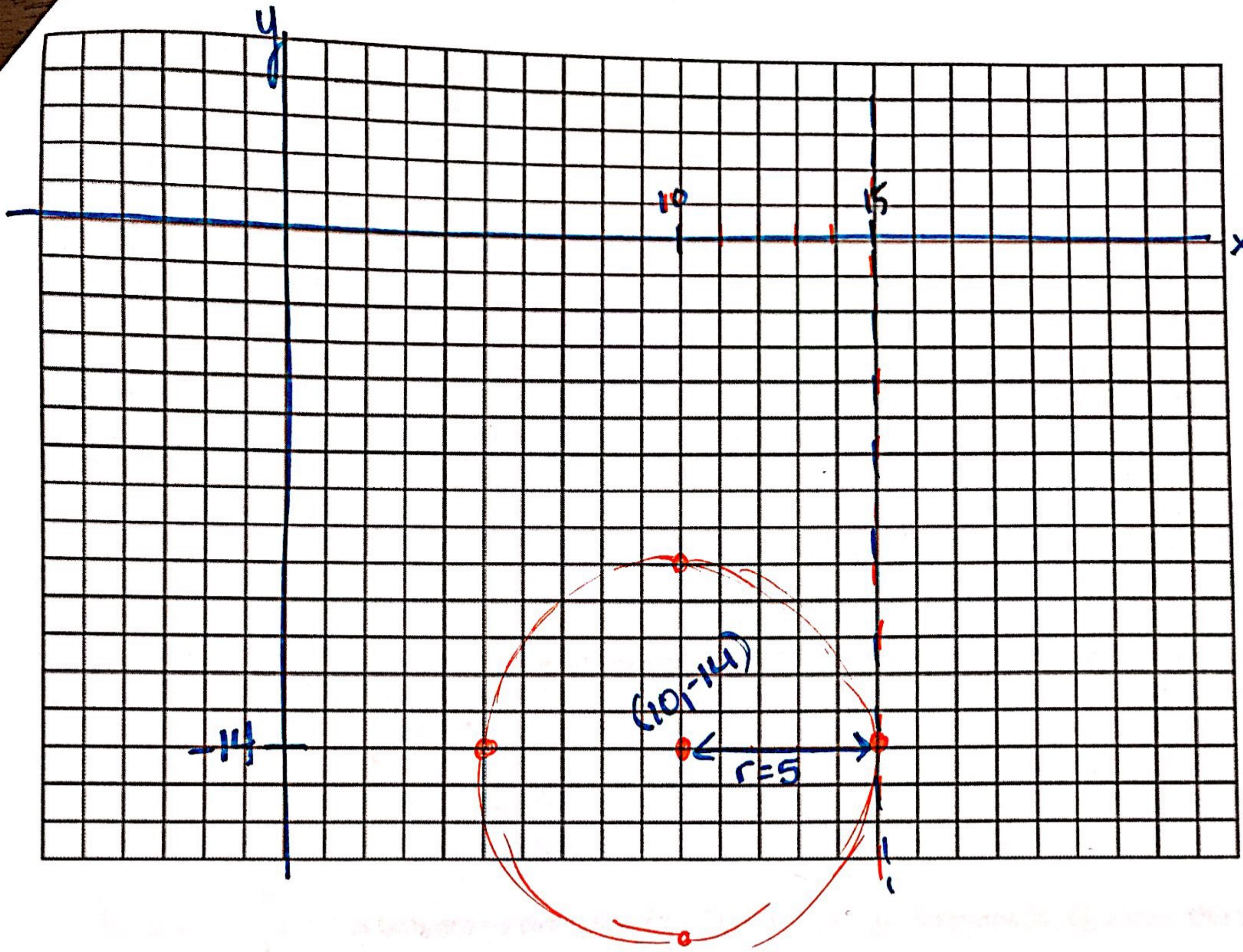
* see graph .... $r=5$

$(x-10)^2+(y+14)^2=25$

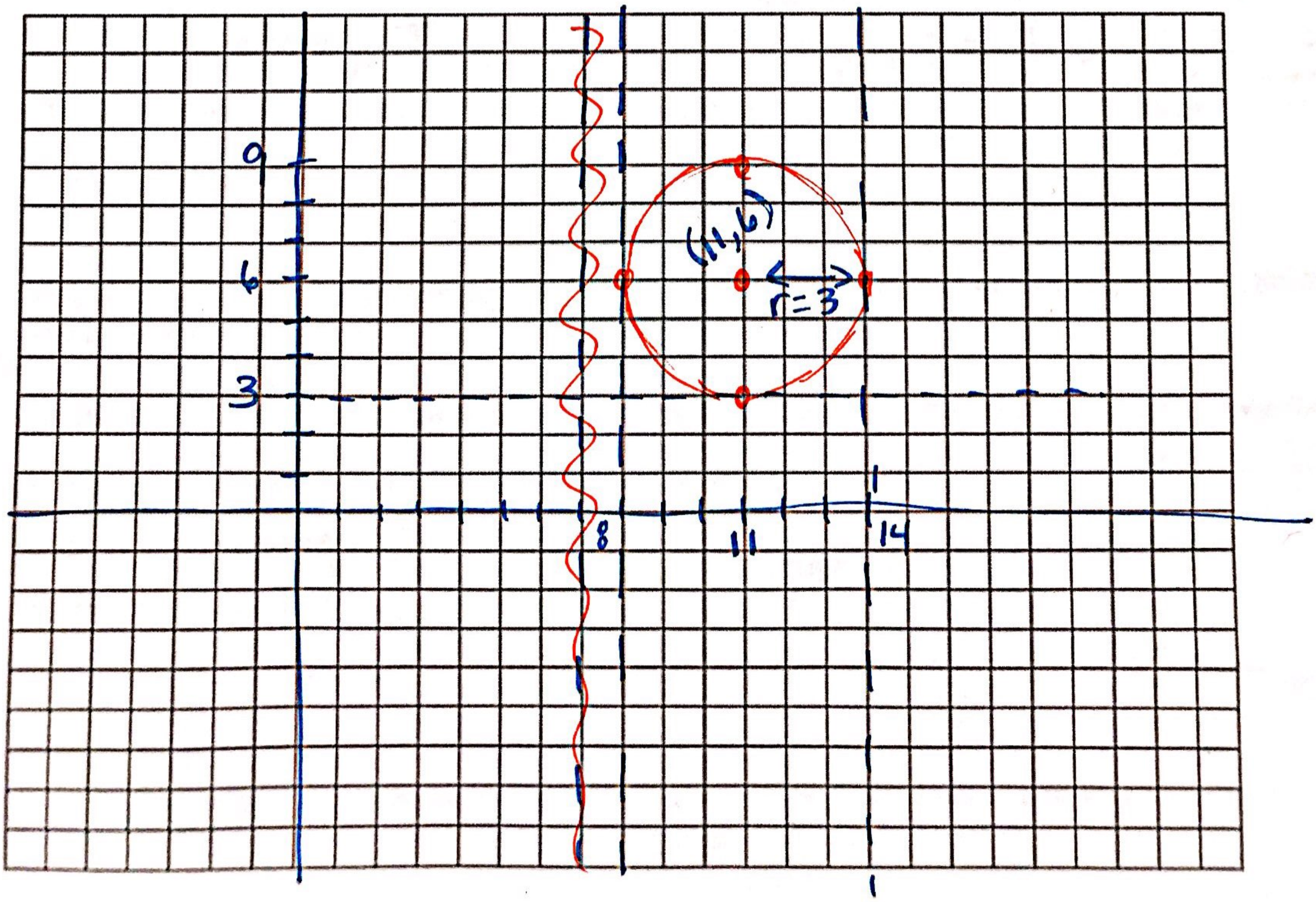
9. **Write the equation of the circle whose center lies in the 1st quadrant and is tangent to the lines x = 8, y = 3, and x = 14.**

* see graph .... $r=3$; center $(11,6)$

$(x-11)^2+(y-6)^2=9$



y
x
10
15
-14
(10,-14)
r=5



9
6
3
8
11
14
(11,6)
r=3