Conics Worksheet 3: Hyperbolas Name: Key

I. Write each of the following equations in graphing form (if not in that form already) and give the key information (center, vertices, foci and asymptotes).

1) $\frac{x^2}{25} - \frac{y^2}{9} = 1$

center: $(0,0)$

$\leftrightarrow 5$ $\updownarrow 3$

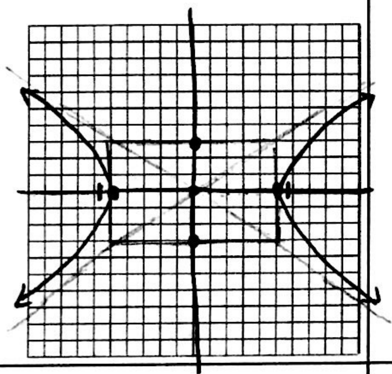
$c = \sqrt{25+9}$

$c = \sqrt{34} \approx 5.8$

V: $(\pm 5, 0)$

F: $(\pm\sqrt{34}, 0)$

A: $y = \pm\frac{3}{5}x$



2) $\frac{(y+2)^2}{4} - \frac{x^2}{25} = 1$

center: $(0,-2)$

$\leftrightarrow 5$ $\updownarrow 2$

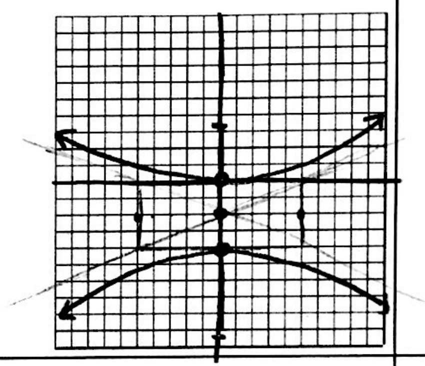
$c = \sqrt{4+25}$

$c = \sqrt{29} \approx 5.4$

V: $(0,0)$ & $(0,-4)$

F: $(0, -2 \pm \sqrt{29})$

A: $y = \pm\frac{2}{5}(x) - 2$



3) $\frac{x^2}{1} - \frac{y^2}{9} = 1$

center $(0,0)$

$\leftrightarrow 1$ $\updownarrow 3$

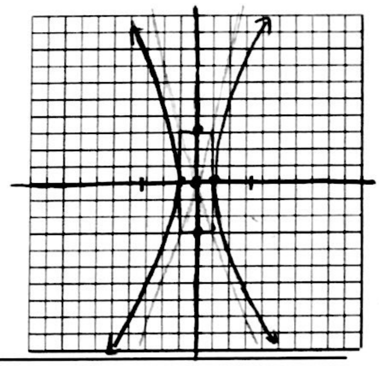
$c = \sqrt{1+9}$

$c = \sqrt{10} \approx 3.2$

V: $(\pm 1, 0)$

F: $(\pm\sqrt{10}, 0)$

A $= \pm\frac{3}{1}x = \pm 3x$



4) $\frac{16y^2 - 9x^2 = 144}{144}$

$\frac{y^2}{9} - \frac{x^2}{16} = 1$

center $= (0,0)$

$\leftrightarrow 4$ $\updownarrow 3$

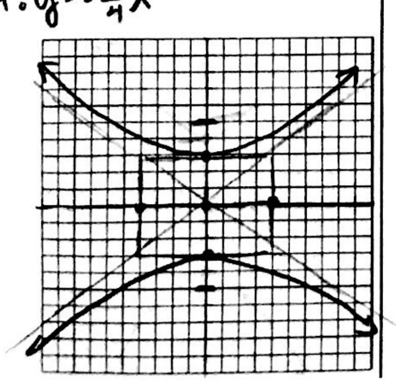
$c = \sqrt{16+9}$

$c = \sqrt{25} = 5$

V: $(0, \pm 3)$

F: $(0, \pm 5)$

A: $y = \pm\frac{3}{4}x$



5) $\frac{(x-4)^2 - (y+2)^2 = 16}{16}$

$\frac{(x-4)^2}{16} - \frac{(y+2)^2}{16} = 1$

center: $(4,-2)$

$\leftrightarrow 4$ $\updownarrow 4$

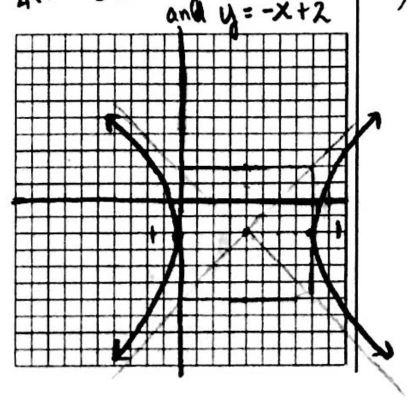
$c = \sqrt{16+16}$

$c = \sqrt{32} = 4\sqrt{2} \approx 5.7$

V: $(8,-2)$ & $(0,-2)$

F: $(4 \pm 4\sqrt{2}, -2)$

A: $y = \pm\frac{4}{4}(x-4) - 2 \Rightarrow y = x - 6$ and $y = -x + 2$



6) $y^2 - x^2 + 4y - 21 = 0$

$(y^2 + 4y + 4) - x^2 = 21 + 4$

$(y+2)^2 - x^2 = 25$

$\frac{(y+2)^2}{25} - \frac{x^2}{25} = 1$

center $(0,-2)$

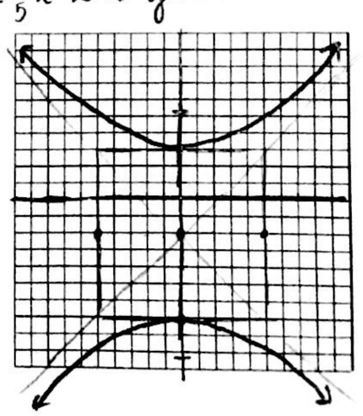
$\leftrightarrow 5$ $\updownarrow 5$

$c = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \approx 7.1$

V: $(0,3)$ & $(0,7)$

F: $(0, -2 \pm 5\sqrt{2})$

A: $y = \pm\frac{5}{5}x - 2 \Rightarrow y = \pm x - 2$



II. Convert each equation to graphing form. Give the key information.

7) $x^2 - y^2 - 6x = 0$

$x^2 - 6x - y^2 = 0$

$(x^2 - 6x + 9) - y^2 = 0 + 9$

$\frac{(x-3)^2 - y^2}{9} = \frac{9}{9}$

$\boxed{\frac{(x-3)^2}{9} - \frac{y^2}{9} = 1}$

center: $(3,0)$

V: $(6,0)$ & $(0,0)$

$c = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

F: $(3 \pm 3\sqrt{2}, 0)$

A: $y = \pm\frac{3}{3}(x-3)+0$

$y = \pm(x-3)$

$y = x-3$ $\quad y = -x+3$

8) $16x^2 - y^2 + 32x + 6y + 39 = 0$

$16(x^2+2x+1) - (y^2-6y+9) = -39+16-9$

$\frac{16(x+1)^2 - (y-3)^2}{-32} = \frac{-32}{-32}$

$\boxed{\frac{-(x+1)^2}{2} + \frac{(y-3)^2}{32} = 1}$

$c = \sqrt{2+32} = \sqrt{34}$

center: $(-1,3)$

V: $(-1, 3 \pm \sqrt{32})$

or $(-1, 3 \pm 4\sqrt{2})$

F: $(-1, 3 \pm \sqrt{34})$

A: $y = \pm\frac{\sqrt{32}}{\sqrt{2}}(x+1)-3 \Rightarrow y = \pm 4(x+1)-3$

$y = 4x+1$ $\quad y = -4x-4$

9) $4y^2 - 25x^2 - 32y + 164 = 0$

$4(y^2 - 8y + 16) - 25x^2 = -164 + 64$

$\frac{4(y-4)^2 - 25x^2}{-100} = \frac{-100}{-100}$

$\boxed{\frac{-(y-4)^2}{25} + \frac{x^2}{4} = 1}$

center: $(0,4)$

V: $(\pm 2, 4)$

$c = \sqrt{25+4} = \sqrt{29}$

F: $(\pm\sqrt{29}, 4)$

A: $y = \pm\frac{5}{4}x + 4$

10) $9y^2 - 4x^2 - 18y + 24x - 63 = 0$

$9(y^2 - 2y + 1) - 4(x^2 - 6x + 9) = 63 + 9 - 36$

$\frac{9(y-1)^2 - 4(x-3)^2}{36} = \frac{36}{36}$

$\boxed{\frac{(y-1)^2}{4} - \frac{(x-3)^2}{9} = 1}$

center: $(3,1)$

V: $(3,3)$ and $(3,-1)$

$c = \sqrt{4+9} = \sqrt{13}$

F: $(3, 1 \pm \sqrt{13})$

A: $y = \pm\frac{2}{3}(x-3)+1$

$y = \frac{2}{3}x - 1$ and $y = -\frac{2}{3}x + 3$

III. Write the equation of the hyperbola in graphing form from the given information.

11) Vertices at (2, 0) and (-2, 0); foci at (3, 0) and (-3, 0)

center $(0,0)$

$3 = \sqrt{4+b^2}$

$9 = 4 + b^2 \Rightarrow b^2 = 5$

$\boxed{-\frac{y^2}{5} + \frac{x^2}{4} = 1}$

12) Vertices at (9, -3) and (-5, -3); foci at $(2 \pm \sqrt{53}, -3)$

c

$\boxed{\frac{(x-2)^2}{49} - \frac{(y+3)^2}{4} = 1}$

$\sqrt{53} = \sqrt{49+b^2}$

$53 = 49 + b^2$

$4 = b^2$

$(2,-3)$

13) Center at the origin, vertex at (-3, 0) and an asymptote with the equation $y = \frac{5}{3}x$

$\boxed{\frac{x^2}{9} - \frac{y^2}{25} = 1}$

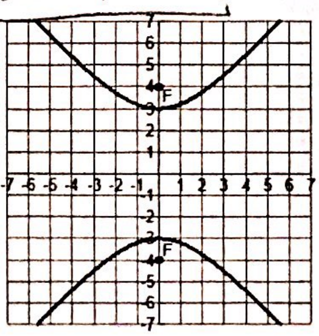
$m = \frac{5}{3} = \frac{b}{a} \Rightarrow b = 5$

14) Vertices at (0, 6) and (0, -6); and an asymptote with the equation y = 3x

$\boxed{\frac{y^2}{36} - \frac{x^2}{4} = 1}$

$m = 3 = \frac{a}{b} \Rightarrow 3 = \frac{6}{b} \Rightarrow b = 2$

15) From the graph: a)



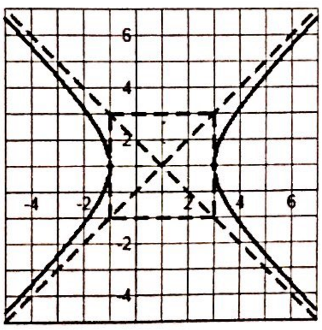
$\boxed{\frac{y^2}{9} - \frac{x^2}{7} = 1}$

$4 = \sqrt{9+b^2}$

$16 = 9 + b^2$

$7 = b^2$

b)



$\boxed{\frac{(x-1)^2}{4} - \frac{(y-1)^2}{4} = 1}$