

Even/Odd Identities

$$\text{Even: } \cos(-\theta) = \cos(\theta) \quad \sec(-\theta) = \sec(\theta)$$

$$\text{Odd: } \sin(-\theta) = -\sin(\theta) \quad \csc(-\theta) = -\csc(\theta)$$

$$\text{Odd: } \tan(-\theta) = -\tan(\theta) \quad \cot(-\theta) = -\cot(\theta)$$

$$\text{ex: } \frac{\cos(-\theta)}{1 + \sin(-\theta)} = \sec\theta + \tan\theta$$

$$\frac{(1 + \sin\theta) \cos(\theta)}{(1 + \sin\theta)(1 - \sin(\theta))} = \sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\frac{(1 + \sin\theta) \cos\theta}{1 - \sin^2\theta} = \sec\theta + \tan\theta$$

$$\frac{(1 + \sin\theta) \cancel{\cos\theta}}{\cancel{\cos\theta}} = \sec\theta + \tan\theta$$

$$\frac{1 + \sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

$$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

$$\text{ex: } \frac{\sec(-x)}{\csc(-x)} = -\tan x$$

$$\frac{\sec(x)}{-\csc(x)} = -\tan x$$

$$\frac{1}{\cos x} \cdot \frac{-\sin x}{1} = -\tan x$$

Co-function Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta \quad \csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

$$\text{ex: } \frac{\csc\left(\frac{\pi}{2} - x\right)}{\tan\left(\frac{\pi}{2} - x\right)} = \frac{\sin x}{\cos^2 x}$$

$$\frac{\sec x}{\cot x} = \frac{\sin x}{\cos^2 x}$$

$$\frac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x}} = \frac{\sin x}{\cos^2 x}$$

$$\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \frac{\sin x}{\cos^2 x}$$

Misc. Identities

$$\text{ex: } \csc^4 x - 2 \csc^2 x + 1 = \cot^4 x$$

$$(\csc^2 x - 1)(\csc^2 x - 1) = \cot^4 x$$

$$\cot^2 x \cdot \cot^2 x = \cot^4 x$$

$$* x^4 - 2x^2 + 1 = (x^2 - 1)(x^2 - 1)$$

$$\text{ex: } \sec^4 \theta - \tan^4 \theta = 1 + 2 \tan^2 \theta$$

$$(\sec^2 \theta + \tan^2 \theta) \underbrace{(\sec^2 \theta - \tan^2 \theta)} = 1 + 2 \tan^2 \theta$$

$$(1 + \tan^2 \theta + \tan^2 \theta) \cdot 1 = 1 + 2 \tan^2 \theta$$

$$1 + 2 \tan^2 \theta = 1 + 2 \tan^2 \theta$$