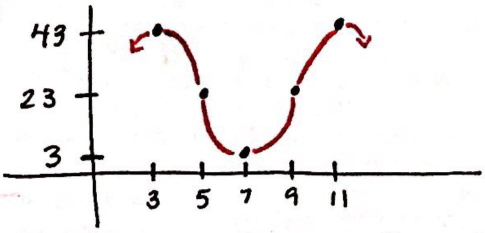
Accelerated Precalculus
Sinusoidal Modeling

Name Key

1. **Ferris Wheel Problem** As you ride the Ferris wheel, your distance from the ground varies sinusoidally with time. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 feet.
   a. Sketch a graph of this sinusoid.
   b. Write an equation of the sinusoid.
   c. What is the lowest you go as the Ferris wheel turns, and why is this number greater than zero?
   d. Predict your height above the ground when
      i. $t = 6$
      ii. $t = 9$
      iii. $t = 0$
   e. ~~What is the value of t the second time you are 18 feet above the ground?~~

ⓐ



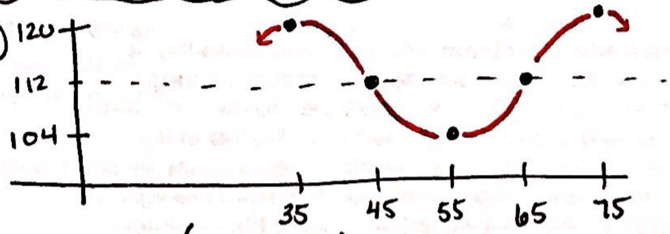
ⓑ $y = 20\cos\left[\frac{\pi}{4}(x-3)\right] + 23$

ⓒ 3 feet

ⓓ $t=6$ $h \approx 8.86$ ft.
$t=9$ $h = 23$ ft
$t=0$ $h \approx 8.86$ ft

2. **Extraterrestrial Being Problem** Researchers find a being from an alien planet. Its body temperature is varying sinusoidally with time. 35 minutes after they start timing, it reaches a high of 120° F. 20 minutes after that it reaches its next low 104° F.
   a. Sketch a graph of this sinusoid.
   b. Write an equation expressing temperature in terms of minutes since they started timing.
   c. What was its temperature when they first started timing?
   d. ~~Find the first three times after they started timing at which the temperature was 114° F.~~

ⓐ

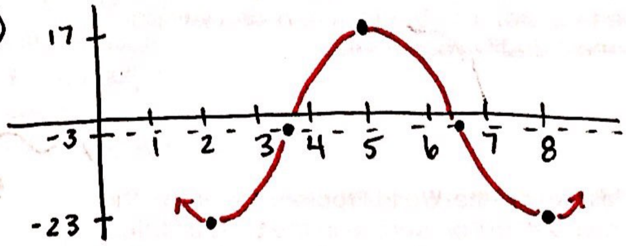


ⓑ $y = 8\cos\left(\frac{\pi}{20}(x-35)\right) + 112$

ⓒ $x=0$ $y = 117.66°$

3. **Tarzan Problem** Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternatively over land and water. Jane decides to model mathematically his motion and starts her stopwatch. Let t be the number of seconds the stopwatch reads and y be the number of meters Tarzan is from the river bank. Assume that y varies sinusoidally with t, and that y is positive when Tarzan is over water and negative when he is over land.
Jane finds that when $t = 2$, Tarzan is at one end of his swing, where $y = -23$. She finds that when $t = 5$ he reaches the other end of his swing and $y = 17$.
   a. Sketch a graph of this sinusoid.
   b. Write an equation expressing Tarzan's distance from the riverbank in terms of t.
   c. Predict y when
      i. $t = 2.8$
      ii. $t = 6.3$
      iii. $t = 15$
   d. Where was Tarzan when Jane started her stopwatch?
   e. ~~Find the least positive value of t for which Tarzan is directly over the river bank (i.e., $y = 0$)~~

ⓐ



ⓑ $y = -20\cos\left(\frac{\pi}{3}(x-2)\right) - 3$

ⓒ $t = 2.8$ $y = -16.38$ meters (land)
$t = 6.3$ $y = 1.16$ meters (water)
$t = 15$ $y = -13$ m (land)

ⓓ $t = 0$ $y = 7$ m (water)

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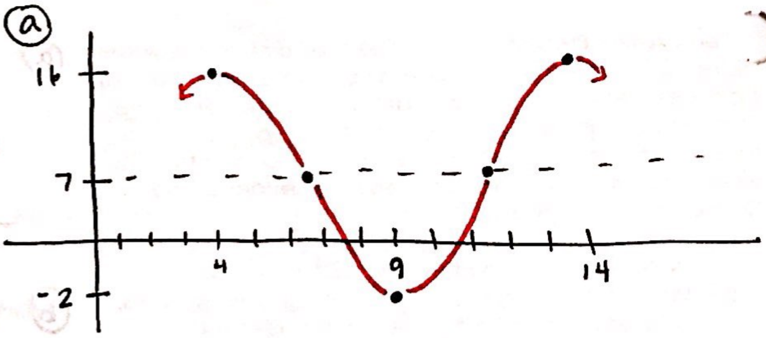
4. **Steamboat Problem** Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d, from the water's surface was a sinusoidal function of time. When his stopwatch read 4 s, the point was at its highest, 16 ft above the waters surface. The wheels diameter was 18 ft, and it completed a revolution every 10 s.
   a. Sketch a graph of the sinusoid.
   b. Writhe the equation of the sinusoid.
   c. How far above the surface was the point when Mark's stopwatch read:
      i. 5 s
      ii. 17 s
   ~~e. What is the first positive value of time at which the point was at the water's surface? At that time, was it going into or coming out of the water? Explain.~~

(a)



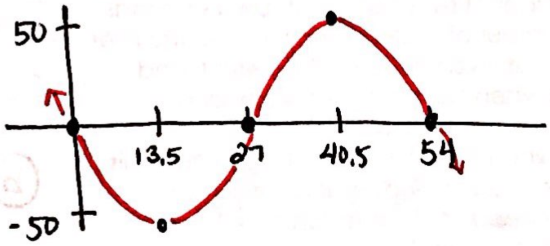
(b) $y = 9\cos\left(\frac{\pi}{5}(x-4)\right) + 7$

(c) $x = 5$ sec $y = 14.28$ ft

$x = 17$ sec $y = 4.22$ ft

5. ~~**Buried Treasure Problem** You seek a treasure that is buried in the side of a mountain. The mountain range has a sinusoidal cross section. The valley to the left is filled with water to a depth of 50 meters and the top of the range is 150 meters above the water level. You set up an x-axis at water level and a y-axis 200 meters to the right of the deepest part of the water. The top of the mountain is at x = 400 meters.~~
   ~~a. Write an equation expressing y in terms of x for points on the surface of the mountain.~~
   ~~b. Show by calculation that this sinusoid contains the origin (0, 0).~~
   ~~c. The treasure is located within the mountain at the point (x, y) = (130, 40). (This point is not on the graph!) Which would be a shorter way to dig to the treasure, a horizontal tunnel or a vertical tunnel? Justify your answer.~~

6. **Shock-Felt-Round-the-World Problem** Suppose that one day all 290 million people in the United States climb up on tables. At time *t* = 0, we all jump off. The resulting shock as we hit the earth's surface will start the entire earth vibrating in such a way that its surface first moves down from its normal position and then moves up an equal distance above its normal position. The displacement y of the surface is a sinusoidal function of time with a period of about 54 minutes. Assume the amplitude is 50 meters.
   a. At what time will the first maximum (i.e., the greatest distance above the normal position) occur?
   b. Write an equation expressing displacement in terms of time elapsed since the people jumped.
   c. Predict the displacement when *t* = 21.
   ~~d. What are the first three times at which the displacement is –37 meters?~~



(a) 40.5 sec

(b) $y = -50\sin\left(\frac{\pi}{27}x\right)$

(c) $t = 21$ $y = -32.14$ m (below normal)