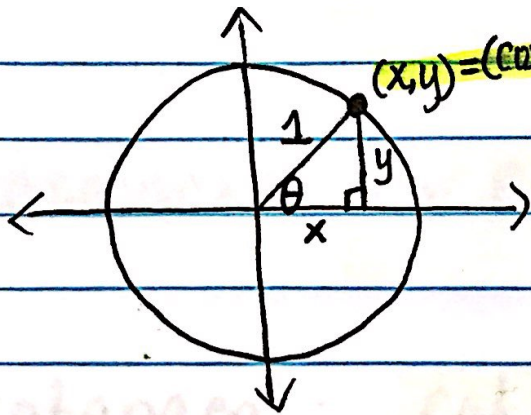


# Sine, Cosine, and Tangent

Recall: SOH CAH TOA



$$(x, y) = (\cos \theta, \sin \theta)$$

$$\sin \theta = \frac{y}{1} \Rightarrow y = \sin \theta$$

$$\cos \theta = \frac{x}{1} \Rightarrow x = \cos \theta$$

$$\tan \theta = \frac{y}{x} \Rightarrow \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\text{ex: } \cos 30^\circ = \frac{\sqrt{3}}{2} ; \sin 30^\circ = \frac{1}{2} ; \tan 30^\circ = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$\text{ex: } \cos^{-\frac{\pi}{4}} = \frac{\sqrt{2}}{2} ; \sin^{-\frac{\pi}{4}} = -\frac{\sqrt{2}}{2} ; \tan^{-\frac{\pi}{4}} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$$

$$\tan^{-\frac{\pi}{4}} = -1$$

$$\text{ex: } \cos \frac{\pi}{2} = 0 ; \sin \frac{\pi}{2} = 1 ; \tan \frac{\pi}{2} = \frac{1}{0} = \text{undefined}$$

$$\text{ex: } \cos \frac{8\pi}{3} = -\frac{1}{2} ; \sin \frac{8\pi}{3} = \frac{\sqrt{3}}{2} ; \tan \frac{8\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\sqrt{3} \cdot 2}{-1} = -\sqrt{3}$$

$$\frac{8\pi}{3} = \left( \frac{2\pi}{3} \right)$$

↖  
coterminal

# The Reciprocal Functions

$$\text{Secant: } \sec \theta = \frac{1}{\cos \theta}$$

$$\text{cosecant: } \csc \theta = \frac{1}{\sin \theta}$$

$$\text{cotangent: } \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$\begin{aligned} \text{ex: } \csc \left( \frac{\pi}{2} \right) &= 1 \\ (0, 1) \sec \left( \frac{\pi}{2} \right) &= \text{und} \\ \cot \left( \frac{\pi}{2} \right) &= 0 \end{aligned}$$

$$\begin{aligned} \text{ex: } \csc \left( \frac{\pi}{6} \right) &= 2 \\ \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) \sec \left( \frac{\pi}{6} \right) &= \frac{2}{\frac{1}{2}} = \frac{2 \cdot \sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} \\ \cot \left( \frac{\pi}{6} \right) &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{ex: } \csc \left( \frac{\pi}{4} \right) &= \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}} = \sqrt{2} \\ \left( \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) \sec \left( \frac{\pi}{4} \right) &= \sqrt{2} \\ \cot \left( \frac{\pi}{4} \right) &= 1 \end{aligned}$$

Mine

Exact Trig Values on the Unit Circle

	$y$	$x$	$\frac{y}{x}$	$\frac{1}{y}$	$\frac{1}{x}$	$\frac{x}{y}$
$\theta$ (radians)	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
$(1, 0)$	0	1	0	und	1	und
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$(\frac{1}{2}, \frac{\sqrt{3}}{2})$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$(0, 1)$	1	0	Und	1	und	0
$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\frac{\sqrt{3}}{3}$
$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$
$(-1, 0)$	0	-1	0	und	-1	und
$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2	$\frac{\sqrt{3}}{3}$
$(0, -1)$	-1	0	und	-1	und	0
$(\frac{1}{2}, -\frac{\sqrt{3}}{2})$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\frac{\sqrt{3}}{3}$
$(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
$(\frac{\sqrt{3}}{2}, -\frac{1}{2})$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$