Solving Systems of Equations Using Inverse Matrices

Step 1 **Write the system as a matrix equation $AX = B$. The matrix A is the** coefficient **matrix, X is the matrix of** variables**, and B is the matrix of** constants**.**

A **Coefficient Matrix:** contains the coefficients of the variables. Each row represents one equation. Each column represents one variable

X **Variable Matrix:** a column matrix containing the variables

B **Constant Matrix:** a column matrix containing the constant values

Step 2 **Find the inverse of matrix A.**

Step 3 **Multiply each side of $AX = B$ by A^{-1} on the** left **to find the solution $X = A^{-1}B$.**

$A^{-1} \cdot A \cdot X = A^{-1} \cdot B$

$X = A^{-1} \cdot B$

Example 1: Use an inverse matrix to solve the linear system

$$2x + 3y = 15$$
$$x - 2y = -17$$

If A does not have an inverse, then the system has either no solution or infinitely many solutions.

① $A \cdot X = B$

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -17 \end{bmatrix}$$

② $$A^{-1} = \frac{1}{-4-3}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2/7 & 3/7 \\ 1/7 & -2/7 \end{bmatrix} = A^{-1}$$

③ $X = A^{-1} \cdot B$

$$X = \begin{bmatrix} 2/7 & 3/7 \\ 1/7 & -2/7 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ -17 \end{bmatrix} = \begin{bmatrix} 30/7 + -51/7 \\ 15/7 + 34/7 \end{bmatrix} = \begin{bmatrix} -3 \\ 7 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

(2)×2   2×(1)

Example 2 – Use a calculator and Inverse matrix to solve the system of linear equations.

$$2x + 3y + z = -1$$
$$3x + 3y + z = 1$$
$$2x + 4y + z = -2$$

$$A \cdot X = B$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$