

Equations of Circles in General Form NOTES

If the equation of the circle is in general form: $x^2 + y^2 + Cx + Dy + E = 0$,

we must complete the square to get it in standard form: $(x - h)^2 + (y - k)^2 = r^2$

Example 1

$$x^2 + y^2 + 16x - 22y - 20 = 0 \quad +20$$

$$(x^2 + 16x + \underline{64}) + (y^2 - 22y + \underline{121}) = \underline{20 + 64 + 121}$$

$$\boxed{(x + 8)^2 + (y - 11)^2 = 205}$$

Center $(-8, 11)$ $r = \sqrt{205} \approx 14.3$

Handwritten notes for Example 1:
 $16 \div 2 = 8$
 $8^2 = 64$
 $-22 \div 2 = -11$
 $(-11)^2 = 121$

Steps to rewriting the equation in standard form:

First, prepare the terms:

- ✓ Group the x-terms and leave a space
- ✓ Group the y-terms and leave a space
- ✓ Move the constant and leave 2 spaces.

Then, complete the square:

- ✓ $\frac{1}{2}$ the linear term and square it
- ✓ Add that number to both sides
- ✓ Do this for both x and y
- ✓ Factor and simplify

Example 2

$$x^2 + y^2 - 12x + 8y + 32 = 0 \quad -32$$

$$(x^2 - 12x + \underline{36}) + (y^2 + 8y + \underline{16}) = \underline{-32 + 36 + 16}$$

$$\boxed{(x - 6)^2 + (y + 4)^2 = 20}$$

Center $(6, -4)$ $r = 2\sqrt{5} \approx 4.5$

Handwritten notes for Example 2:
 $-12 \div 2 = -6$
 $(-6)^2 = 36$
 $8 \div 2 = 4$
 $(4)^2 = 16$

Example 3

$$2x^2 + 2y^2 + 8x - 12y + 8 = 0$$

2

$$x^2 + y^2 + 4x - 6y + 4 = 0$$
$$(x^2 + 4x + \overset{-4}{4}) + (y^2 - 6y + \overset{-4}{9}) = \underbrace{-4 + \frac{4}{1} + \frac{9}{1}}_9$$

$$(x+2)^2 + (y-3)^2 = 9$$

center = (-2, 3) r = 3

Example 4

$$x^2 + y^2 - 10x - 11 = 0$$

$$(x^2 - 10x + \overset{+25}{25}) + y^2 = \overset{+11}{11} + \overset{+25}{25}$$

$$(x-5)^2 + y^2 = 36$$

center (5, 0) radius = 6

Example 5

$$x^2 + y^2 - 8x - 6y + 16 = 0$$