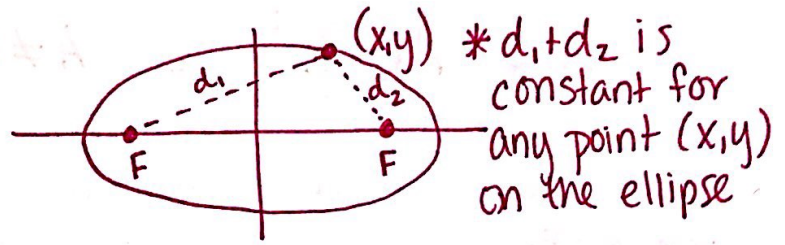


Conic Sections: Ellipses Notes



$a > b$

**KeyConcept Standard Forms of Equations for Ellipses**

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Orientation: horizontal major axis  
 Center:  $(h, k)$   
 Foci:  $(h \pm c, k)$   
 Vertices:  $(h \pm a, k)$   
 Co-vertices:  $(h, k \pm b)$   
 Major axis:  $y = k$   
 Minor axis:  $x = h$   
 a, b, c relationship:  $c^2 = a^2 - b^2$  or  $c = \sqrt{a^2 - b^2}$

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

Orientation: vertical major axis  
 Center:  $(h, k)$   
 Foci:  $(h, k \pm c)$   
 Vertices:  $(h, k \pm a)$   
 Co-vertices:  $(h \pm b, k)$   
 Major axis:  $x = h$   
 Minor axis:  $y = k$   
 a, b, c relationship:  $c^2 = a^2 - b^2$  or  $c = \sqrt{a^2 - b^2}$

\* The foci always lie on the major axis "c" units from the center.

Graph the ellipse given by each equation. Identify Center, Foci, Vertices, Co-vertices, Major/Minor Axis

Example 1:  $\frac{(x-3)^2}{36} + \frac{(y+1)^2}{9} = 1$   
 $a=6$       $b=3$

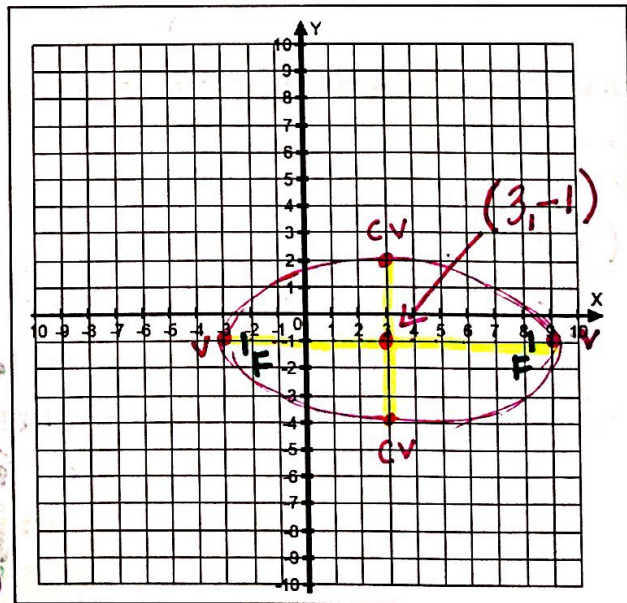
center  $(3, -1)$

V:  $(-3, -1)$  and  $(9, -1)$   
 CV:  $(3, 2)$  and  $(3, -4)$

$c = \sqrt{a^2 - b^2}$   
 $c = \sqrt{36 - 9}$   
 $c = \sqrt{27}$   
 $c = 3\sqrt{3} \approx 5.2$

F:  $(3 + 3\sqrt{3}, -1) \approx (8.2, -1)$   
 $(3 - 3\sqrt{3}, -1) \approx (-2.2, -1)$

Major Axis:  $y = -1$   
 Minor Axis:  $x = 3$



Ellipse  $Ax^2 + By^2 + \dots$

$A \neq B$  and  $A+B$  have the same sign

Example 2:  $4x^2 + y^2 - 24x + 4y + 24 = 0$

Write in S.F.

$$4x^2 - 24x + y^2 + 4y = -24$$

$$4(x^2 - 6x + 9) + (y^2 + 4y + 4) = -24 + 36 + 4$$

$$\frac{4(x-3)^2 + (y+2)^2}{16} = 1$$

$$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{16} = 1$$

$b=2 \leftarrow$        $a=4 \uparrow$

center:  $(3, -2)$

V:  $(3, 2)$  and  $(3, -6)$

CV:  $(1, -2)$  and  $(5, -2)$

$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{16 - 4}$$

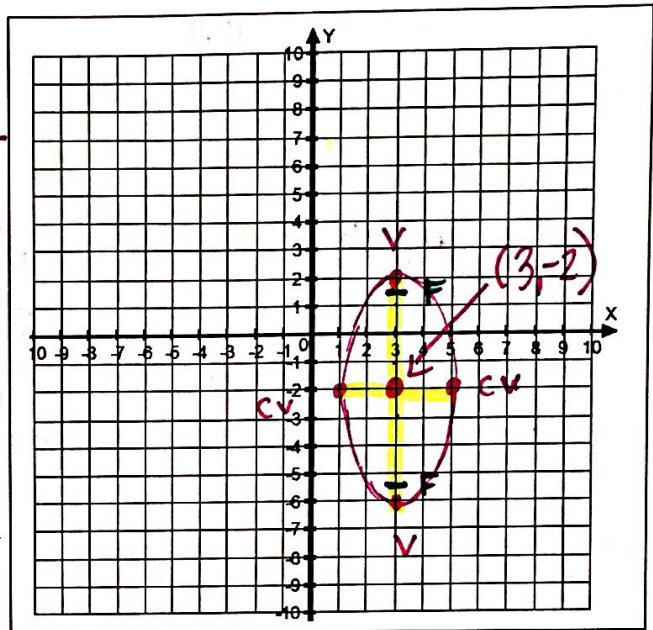
$$c = \sqrt{12}$$

$$c = 2\sqrt{3} \approx 3.5$$

Foci:  $(3, -2 + 2\sqrt{3}) \approx (3, 1.5)$   
 $(3, -2 - 2\sqrt{3}) \approx (3, -5.5)$

Major Axis:  $x=3$

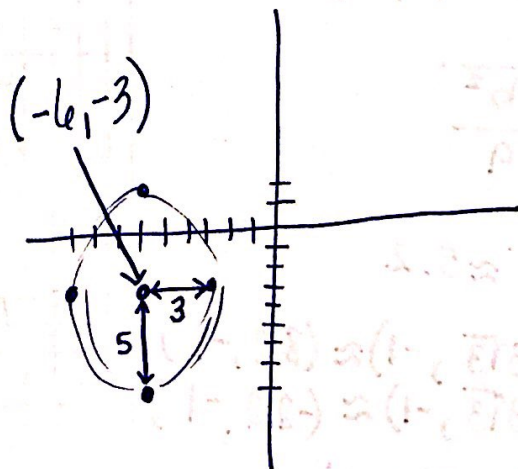
Minor Axis:  $y=-2$



Write an equation for an ellipse with each set of characteristics

Example 3: major axis from  $(-6, 2)$  to  $(-6, -8)$ ; minor axis from  $(-3, -3)$  to  $(-9, -3)$

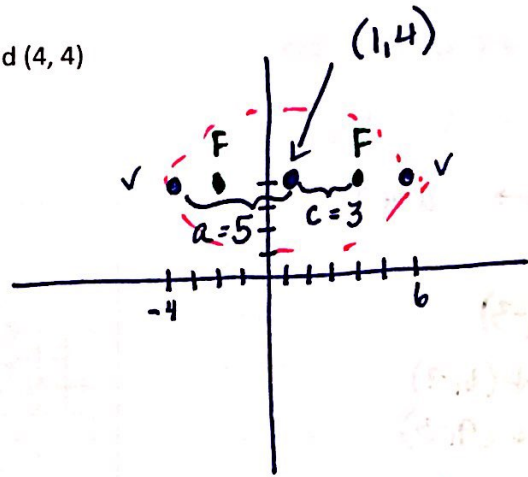
$$\frac{(x+6)^2}{9} + \frac{(y+3)^2}{25} = 1$$





Example 4: vertices at (-4, 4) and (6, 4); foci at (-2, 4) and (4, 4)

$$\frac{(x-1)^2}{25} + \frac{(y-4)^2}{16} = 1$$



$$c = \sqrt{a^2 - b^2}$$

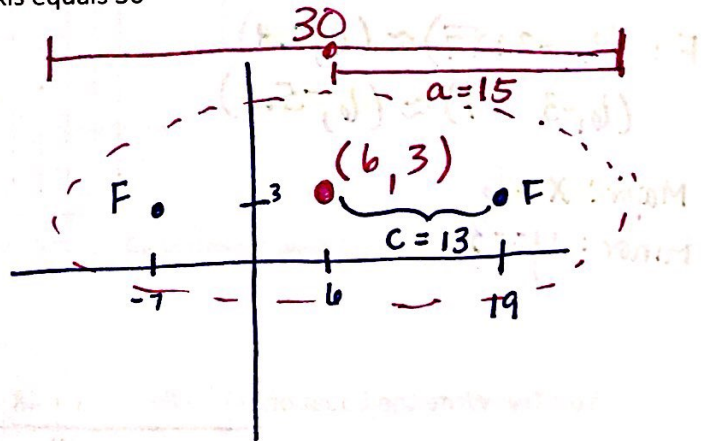
$$(3)^2 = (\sqrt{25 - b^2})^2$$

$$9 = 25 - b^2$$

$$\begin{array}{r} -25 \\ -25 \\ \hline +16 = +b^2 \end{array}$$

Example 5: foci at (19, 3) and (-7, 3); length of major axis equals 30

$$\frac{(x-6)^2}{225} + \frac{(y-3)^2}{56} = 1$$



$$c = \sqrt{a^2 - b^2}$$

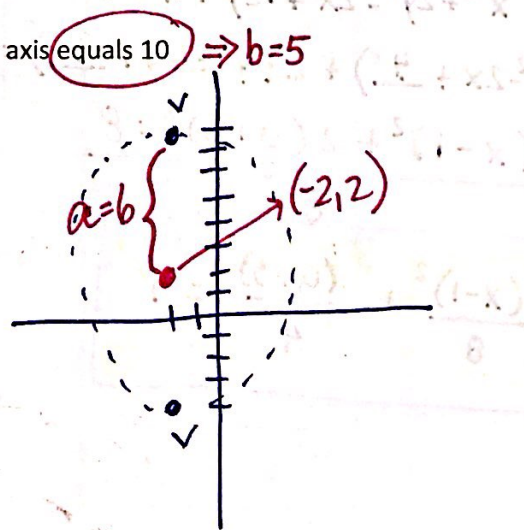
$$(13)^2 = (\sqrt{225 - b^2})^2$$

$$169 = 225 - b^2$$

$$\begin{array}{r} -225 \\ -225 \\ \hline +56 = +b^2 \end{array}$$

Example 6: vertices at (-2, -4) and (-2, 8); length of minor axis equals 10  $\Rightarrow b=5$

$$\frac{(x+2)^2}{25} + \frac{(y-2)^2}{36} = 1$$



You Try: Graph the Ellipse

$$\frac{(x-6)^2}{9} + \frac{(y+3)^2}{16} = 1$$

$b=3 \leftarrow$        $a=4 \downarrow$

center  $(6, -3)$

v:  $(6, 1)$  +  $(6, -7)$

cv:  $(3, -3)$  +  $(9, -3)$

$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{16 - 9} = \sqrt{7}$$

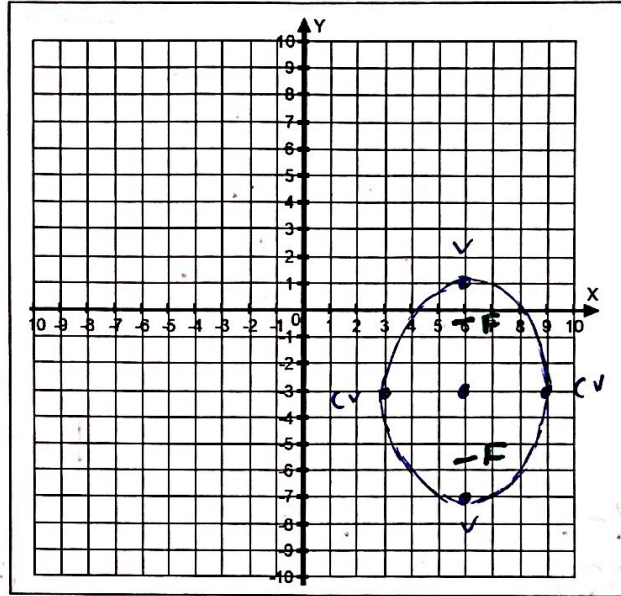
$$c = \sqrt{7} \approx 2.6$$

F:  $(6, -3 + \sqrt{7}) \approx (6, -0.4)$

$(6, -3 - \sqrt{7}) \approx (6, -5.6)$

Major:  $x=6$

Minor:  $y=-3$



You Try: Write the Equation  $4x^2 + 8y^2 - 8x + 48y + 44 = 0$  in Standard Form

4

$$x^2 + 2y^2 - 2x + 12y + 11 = 0$$

$$(x^2 - 2x + \underline{1}) + 2(y^2 + 6y + \underline{9}) = -11 + \underline{1} + \underline{18}$$

$$(x-1)^2 + 2(y+3)^2 = 8$$

8

$$\frac{(x-1)^2}{8} + \frac{(y+3)^2}{4} = 1$$