Verifying Trig Identities (Day 2)

(1) $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

$\tan^2\theta \cos^2\theta = \sin^2\theta$

$\frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta} = \sin^2\theta$

(5) $\cot^2\theta \csc^2\theta - \cot^2\theta = \cot^4\theta$

$\cot^2\theta(\csc^2\theta - 1) = \cot^4\theta$

$\cot^2\theta(\cot^2\theta) = \cot^4\theta$

(2) $\sec^2\theta(1-\cos^2\theta) = \tan^2\theta$

$\sec^2\theta(\sin^2\theta) = \tan^2\theta$

$\frac{1}{\cos^2\theta} \cdot \sin^2\theta = \tan^2\theta$

(6) $\tan\theta \csc^2\theta - \tan\theta = \cot\theta$

$\tan\theta(\csc^2\theta - 1) = \cot\theta$

$\tan\theta(\cot^2\theta) = \cot\theta$

$\frac{1}{\cot\theta} \cdot \frac{\cot^2\theta}{1} = \cot\theta$

(3) $\sin\theta - \sin\theta\cos^2\theta = \sin^3\theta$

$\sin\theta(1-\cos^2\theta) = \sin^3\theta$

$\sin\theta(\sin^2\theta) = \sin^3\theta$

(7) $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

$\frac{\cos\theta}{\cos\theta}\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\frac{\sin\theta}{\sin\theta} = \cot\theta$

$\frac{\cos\theta\sec\theta - \sin^2\theta}{\cos\theta\sin\theta} = \cot\theta$

$\frac{\frac{\cancel{\cos\theta}}{1} \cdot \frac{1}{\cancel{\cos\theta}} - \sin^2\theta}{\cos\theta\sin\theta} = \cot\theta$

$\frac{1-\sin^2\theta}{\cos\theta\sin\theta} = \cot\theta$

$\frac{\cancel{\cos^2\theta}}{\cancel{\cos\theta}\sin\theta} = \cot\theta$

$\frac{\cos\theta}{\sin\theta} = \cot\theta$

(4) $\csc\theta - \cos\theta\cot\theta = \sin\theta$

$\frac{1}{\sin\theta} - \cos\theta \cdot \frac{\cos\theta}{\sin\theta} = \sin\theta$

$\frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta} = \sin\theta$

$\frac{1-\cos^2\theta}{\sin\theta} = \sin\theta$

$\frac{\sin^2\theta}{\sin\theta} \cdot \sin\theta$

(8) $\dfrac{\sin\theta}{1-\cos\theta} + \dfrac{1-\cos\theta}{\sin\theta} = 2\csc\theta$

$\dfrac{\sin\theta \cdot \sin\theta}{\sin\theta(1-\cos\theta)} + \dfrac{(1-\cos\theta)(1-\cos\theta)}{\sin\theta(1-\cos\theta)} = 2\csc\theta$

$\dfrac{\sin^2\theta + (1-\cos\theta)(1-\cos\theta)}{\sin\theta(1-\cos\theta)} = 2\csc\theta$

$\dfrac{\sin^2\theta + 1 - \cos\theta - \cos\theta + \cos^2\theta}{\sin\theta(1-\cos\theta)} = 2\csc\theta$

$\dfrac{\sin^2\theta + \cos^2\theta + 1 - 2\cos\theta}{\sin\theta(1-\cos\theta)} = 2\csc\theta$

$\dfrac{1 + 1 - 2\cos\theta}{\sin\theta(1-\cos\theta)} = 2\csc\theta$

$\dfrac{2-2\cos\theta}{\sin\theta(1-\cos\theta)} = 2\csc\theta$

$\dfrac{2\cancel{(1-\cos\theta)}}{\sin\theta\cancel{(1-\cos\theta)}} = 2\csc\theta$

$\dfrac{2}{\sin\theta} = 2\csc\theta$

* Another way to do this problem is at the end

(9) $\dfrac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$

$\dfrac{(1-\sin\theta)\cos\theta}{(1-\sin\theta)(1+\sin\theta)} + \dfrac{\tan\theta}{1} = \sec\theta$

$\dfrac{(1-\sin\theta)\cos\theta}{1+\sin\theta-\sin\theta-\sin^2\theta} + \tan\theta = \sec\theta$

$\dfrac{(1-\sin\theta)\cos\theta}{1-\sin^2\theta} + \tan\theta = \sec\theta$

$\dfrac{(1-\sin\theta)\cos\theta}{\cos^2\theta} + \tan\theta = \sec\theta$

$\dfrac{1-\sin\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \sec\theta$

$\dfrac{1-\sin\theta+\sin\theta}{\cos\theta} = \sec\theta$

$\dfrac{1}{\cos\theta} = \sec\theta$

(10) Done on last page

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(11) $$\frac{1}{1-\tan^2\theta} + \frac{1}{1-\cot^2\theta} = 1$$

$$\frac{(1-\cot^2\theta)\,1}{(1-\cot^2\theta)(1-\tan^2\theta)} + \frac{1\,(1-\tan^2\theta)}{(1-\cot^2\theta)(1-\tan^2\theta)} = 1$$

$$\frac{1-\cot^2\theta + 1 - \tan^2\theta}{(1-\cot^2\theta)(1-\tan^2\theta)} = 1$$

$$\frac{2-\cot^2\theta - \tan^2\theta}{1-\tan^2\theta - \cot^2\theta + \tan^2\theta\cot^2\theta} = 1$$

$$\frac{2-\cot^2\theta - \tan^2\theta}{1-\tan^2\theta - \cot^2\theta + \frac{\tan^2\theta}{1}\cdot\frac{1}{\tan^2\theta}} = 1$$

$$\frac{2-\cot^2\theta - \tan^2\theta}{1-\tan^2\theta - \cot^2\theta + 1} = 1$$

$$\frac{\cancel{2-\cot^2\theta - \tan^2\theta}}{\cancel{2-\cot^2\theta - \tan^2\theta}} = 1$$

$$1 = 1$$

(12) $\frac{1}{\csc\theta+1} + \frac{1}{\csc\theta-1} = 2\sec^2\theta\sin\theta$

$\frac{(\csc\theta-1)}{(\csc\theta-1)}\frac{1}{(\csc\theta+1)} + \frac{1}{(\csc\theta-1)}\frac{(\csc\theta+1)}{(\csc\theta+1)} = 2\sec^2\theta\sin\theta$

$\frac{\csc\theta-\cancel{1}+\csc\theta+\cancel{1}}{(\csc\theta-1)(\csc\theta+1)} = 2\sec^2\theta\sin\theta$

$\frac{2\csc\theta}{\csc^2\theta+\cancel{\csc\theta}-\cancel{\csc\theta}-1} = 2\sec^2\theta\sin\theta$

$\frac{2\csc\theta}{\csc^2\theta-1} = 2\sec^2\theta\sin\theta$

$\frac{2\csc\theta}{\cot^2\theta} = 2\sec^2\theta\sin\theta$

$\frac{2\cdot\frac{1}{\sin\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = 2\sec^2\theta\sin\theta$

$\frac{2}{\cancel{\sin\theta}}\cdot\frac{\sin^{\cancel{2}}\theta}{\cos^2\theta} = 2\sec^2\theta\sin\theta$

$\frac{2\sin\theta}{\cos^2\theta} = 2\sec^2\theta\sin\theta$ ✓

* another way done at the end

(13) $(\csc\theta-\cot\theta)(\csc\theta+\cot\theta)=1$

$$\csc^2\theta+\cancel{\csc\theta\cot\theta}-\cancel{\cot\theta\csc\theta}-\cot^2\theta=1$$

$$\underbrace{\csc^2\theta}-\cot^2\theta=1$$

$$1+\cancel{\cot^2\theta}-\cancel{\cot^2\theta}=1$$

$$1=1$$

(14) $\cos^4\theta-\sin^4\theta=\cos^2\theta-\sin^2\theta$

$$\underbrace{(\cos^2\theta+\sin^2\theta)}(\cos^2\theta-\sin^2\theta)=\cos^2\theta-\sin^2\theta$$

$$1\cdot(\cos^2\theta-\sin^2\theta)=\cos^2\theta-\sin^2\theta$$

(15) $\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta}=2\sec^2\theta$

$$\frac{(1+\sin\theta)}{(1+\sin\theta)}\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}\frac{(1-\sin\theta)}{(1-\sin\theta)}=2\sec^2\theta$$

$$\frac{1+\cancel{\sin\theta}+1-\cancel{\sin\theta}}{(1+\sin\theta)(1-\sin\theta)}=2\sec^2\theta$$

$$\frac{2}{1-\cancel{\sin\theta}+\cancel{\sin\theta}-\sin^2\theta}=2\sec^2\theta$$

$$\frac{2}{1-\sin^2\theta}=2\sec^2\theta$$

$$\frac{2}{\cos^2\theta}=2\sec^2\theta$$

$$2\sec^2\theta=2\sec^2\theta$$

(16) $\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{\cos\theta}{1-\sin\theta} = 2\sec\theta$

$\dfrac{(1-\sin\theta)\cos\theta}{(1-\sin\theta)(1+\sin\theta)} + \dfrac{\cos\theta\,(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 2\sec\theta$

$\dfrac{(1-\sin\theta)\cos\theta + \cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 2\sec\theta$

$\dfrac{\cos\theta - \cancel{\cos\theta\sin\theta} + \cos\theta + \cancel{\cos\theta\sin\theta}}{1-\sin^2\theta} = 2\sec\theta$

$\dfrac{2\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta} = 2\sec\theta$

$\dfrac{2}{\cos\theta} = 2\sec\theta$

$2\sec\theta = 2\sec\theta$

(17) $\csc^4\theta - \cot^4\theta = 2\cot^2\theta + 1$

$(\csc^2\theta + \cot^2\theta)\underbrace{(\csc^2\theta - \cot^2\theta)}_{1} = 2\cot^2\theta + 1$

$\underbrace{\csc^2\theta} + \cot^2\theta = 2\cot^2\theta + 1$

$1 + \cot^2\theta + \cot^2\theta = 2\cot^2\theta + 1$

$2\cot^2\theta + 1 = 2\cot^2\theta + 1$

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(8) $\frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = 2\csc\theta$

$$\frac{(1+\cos\theta)\sin\theta}{(1+\cos\theta)(1-\cos\theta)} + \frac{1-\cos\theta}{\sin\theta} = 2\csc\theta$$

$$\frac{(1+\cos\theta)\sin\theta}{1-\cos^2\theta} + \frac{1-\cos\theta}{\sin\theta} = 2\csc\theta$$

$$\frac{(1+\cos\theta)\sin\theta}{\sin^2\theta} + \frac{(1-\cos\theta)\sin\theta}{\sin\theta\,\sin\theta} = 2\csc\theta$$

$$\frac{\sin\theta + \sin\theta\cancel{\cos\theta} + \sin\theta - \sin\theta\cancel{\cos\theta}}{\sin^2\theta} = 2\csc\theta$$

$$\frac{2\,\cancel{\sin\theta}}{\sin^{\cancel{2}}\theta} = 2\csc\theta$$

$$\frac{2}{\sin\theta} = 2\csc\theta \quad ✓$$

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(12)

$$\frac{1}{\csc\theta+1}+\frac{1}{\csc\theta-1}=2\sec^2\theta\sin\theta$$

$$\frac{1}{\frac{1}{\sin\theta}+1\frac{\sin\theta}{\sin\theta}}+\frac{1}{\frac{1}{\sin\theta}-1\frac{\sin\theta}{\sin\theta}}=2\sec^2\theta\sin\theta$$

$$\frac{1}{\frac{1+\sin\theta}{\sin\theta}}+\frac{1}{\frac{1-\sin\theta}{\sin\theta}}=2\sec^2\theta\sin\theta$$

$$\frac{(1-\sin\theta)}{(1-\sin\theta)}\frac{\sin\theta}{1+\sin\theta}+\frac{\sin\theta}{1-\sin\theta}\frac{(1+\sin\theta)}{(1+\sin\theta)}=2\sec^2\theta\sin\theta$$

$$\frac{(1-\sin\theta)\sin\theta}{1-\sin^2\theta}+\frac{\sin\theta(1+\sin\theta)}{1-\sin^2\theta}=2\sec^2\theta\sin\theta$$

$$\frac{(1-\sin\theta)\sin\theta+\sin\theta(1+\sin\theta)}{\cos^2\theta}=2\sec^2\theta\sin\theta$$

$$\frac{\sin\theta-\cancel{\sin^2\theta}+\sin\theta+\cancel{\sin^2\theta}}{\cos^2\theta}=\frac{2\sin\theta}{\cos^2\theta}=2\sec^2\theta\sin\theta$$

(10) $$\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = \sin\theta + \cos\theta$$

$$\frac{\sin\theta}{\sin\theta}\;\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{\cos\theta}\;\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} = \sin\theta + \cos\theta$$

$$\frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}} = \sin\theta + \cos\theta$$

$$\frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta}{1}\cdot\frac{\cos\theta}{\cos\theta-\sin\theta} = \sin\theta + \cos\theta$$

$$\frac{\sin^2\theta}{\sin\theta-\cos\theta} + \frac{\cos^2\theta}{\cos\theta-\sin\theta} = \sin\theta + \cos\theta$$

$$\frac{\sin^2\theta}{\sin\theta-\cos\theta} + \frac{\cos^2\theta}{-(-\cos\theta+\sin\theta)} = \sin\theta + \cos\theta$$

$$\frac{\sin^2\theta}{\sin\theta-\cos\theta} - \frac{\cos^2\theta}{\sin\theta-\cos\theta} = \sin\theta + \cos\theta$$

$$\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta} = \sin\theta + \cos\theta$$

$$\frac{(\sin\theta+\cos\theta)\cancel{(\sin\theta-\cos\theta)}}{\cancel{\sin\theta-\cos\theta}} = \sin\theta + \cos\theta$$

$$\sin\theta + \cos\theta = \sin\theta + \cos\theta$$

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