

## Vector Dot Product

Given  $\vec{u} = \langle a, b \rangle$  and  $\vec{v} = \langle c, d \rangle$

$$\vec{u} \bullet \vec{v} = ac + bd$$

ex:  $\vec{u} = 3i - 4j$   $\vec{v} = \langle 2, 2 \rangle$   $\vec{w} = 3i$

$$(a) \vec{u} \bullet \vec{v} = 3 \cdot 2 + (-4) \cdot 2 = -2$$

$$(b) \vec{v} \bullet \vec{w} = 2 \cdot 3 + 2 \cdot 0 = 6$$

## Angle Between Vectors

The angle between vectors  $\vec{u}$  &  $\vec{v}$  can be found by....

$$\theta = \cos^{-1} \left( \frac{\vec{u} \bullet \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|} \right)$$

ex: Find the angle between  $\vec{u} = \langle 3, -2 \rangle$  and  $\vec{v} = \langle -1, 4 \rangle$   
 $\vec{u} \bullet \vec{v} = -11$   $\|\vec{u}\| = \sqrt{13}$   $\|\vec{v}\| = \sqrt{17}$

$$\theta = \cos^{-1} \left( \frac{-11}{\sqrt{13} \cdot \sqrt{17}} \right) \approx 137.7^\circ$$

NOTE: If  $\vec{u} \bullet \vec{v} = 0$ , then the vectors are  $\perp$  aka orthogonal.

Work involves the exertion of energy/force to move an object a certain distance.

To calculate work, find the dot product of the force vector and the displacement vector.

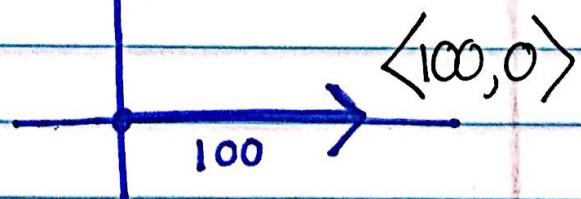
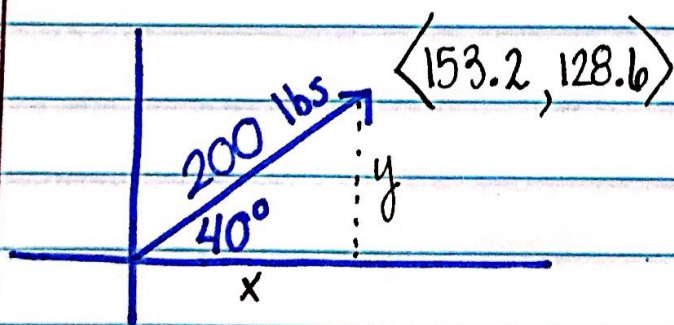
$$W = F \cdot D$$

(units: Newton-Meters or Foot-pounds)

ex: A mover moves a piano on a flat surface 100 ft. Using 200 lbs of force at a  $40^\circ$  angle, how much work is done?

Force vector

Displacement vector



$$W = F \cdot D = \langle 153.2, 128.6 \rangle \cdot \langle 100, 0 \rangle$$

$$\text{Work} = 15,320 \text{ ft-lbs}$$