

Trig Identities

$$\textcircled{1} \tan x \cdot \frac{1}{\cot x} = \tan^2 x$$

$$\tan x \cdot \tan x = \tan^2 x$$

$$\textcircled{2} (1 - \cos x)(1 + \cos x) = \sin^2 x$$

$$1 + \cos x - \cos x - \cos^2 x =$$

$$1 - \cos^2 x =$$

$$\sin^2 x + \cos^2 x - \cos^2 x =$$

$$\sin^2 x$$

$$\textcircled{3} \cos^2 x \csc x \sec x = \cot x$$

$$\cos^2 x \cdot \frac{1}{\sin x} \cdot \frac{1}{\cos x} = \frac{\cos x}{\sin x} = \cot x$$

$$\textcircled{4} (1 - \cos^2 x)(\cot x) = \sin x \cos x$$

$$(\sin^2 x + \cos^2 x - \cos^2 x)(\cot x) =$$

$$\sin^2 x \cdot \cot x = \sin^2 x \cdot \frac{\cos x}{\sin x} = \sin x \cos x$$

$$\textcircled{5} \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\csc x}{\cos x}$$

$$\frac{\sin x \cdot \sin x}{\cos x \cdot \sin x} + \frac{\cos x \cdot \cos x}{\sin x \cdot \cos x} =$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x \sin x} = \frac{\csc x}{\cos x}$$

$$\textcircled{6} \tan x \csc x \cos x = 1$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \cos x = 1$$

$$\textcircled{7} (\sin x - \cos x)^2 = 1 - 2 \sin x \cos x$$

$$\sin^2 x - 2 \sin x \cos x + \cos^2 x = 1 - 2 \sin x \cos x$$

$$\textcircled{8} \frac{\csc x}{\cos x} - \frac{\cos x}{\sin x} = \tan x$$

$$\frac{\frac{1}{\sin x}}{\cos x} - \frac{\cos x}{\sin x} = \frac{1}{\sin x \cos x} - \frac{\cos x \cdot \cos x}{\sin x \cos x} = \frac{1 - \cos^2 x}{\sin x \cos x}$$

$$\frac{\sin^2 x + \cos^2 x - \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x$$

$$\textcircled{9} \csc x - \sin x = \cot x \cos x$$

$$\frac{1}{\sin x} - \frac{\sin x}{1} \cdot \frac{\sin x}{\sin x} = \frac{1 - \sin^2 x}{\sin x} = \frac{\sin^2 x + \cos^2 x - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$$

$$= \frac{\cos x \cdot \cos x}{\sin x} = \cot x \cos x$$

$$\textcircled{10} \cot x (\tan x \sin x + \cos x) = \csc x$$

$$\cot x \tan x \sin x + \cot x \cos x =$$

$$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x} \cdot \sin x + \frac{\cos x}{\sin x} \cdot \cos x =$$

$$\frac{\sin x \cdot \sin x}{\sin x} + \frac{\cos^2 x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$$

$$\textcircled{11} (1 + \sin x)(1 - \sin x) = \cos^2 x$$

$$1 - \sin x + \sin x - \sin^2 x = 1 - \sin^2 x = \sin^2 x + \cos^2 x - \sin^2 x = \cos^2 x$$

$$\textcircled{13} \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$$

$$\frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{\frac{1}{\cos x}} = \sin x \cdot \sin x + \cos x \cdot \cos x$$

$$\frac{1}{\sin x} \cdot \frac{1}{\cos x} = \sin^2 x + \cos^2 x = 1$$

$$\textcircled{14} \frac{1 + \sin x}{\cos x} = \frac{\cos x}{1 - \sin x}$$

$$\frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{\cos x (1 + \sin x)}{1 - \sin^2 x} = \frac{\cos x (1 + \sin x)}{\sin^2 x + \cos^2 x - \sin^2 x}$$

$$\frac{\cos x (1 + \sin x)}{\cos^2 x} = \frac{1 + \sin x}{\cos x}$$

$$(15) \frac{\csc^2 x}{\cot^2 x} = 1 + \tan^2 x$$

$$\frac{\frac{1}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}} = \frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = 1 + \tan^2 x$$

$$(16) \frac{\sec^2 x - 1}{\sin^2 x} = \sec^2 x$$

$$\frac{\sec^2 x - 1}{\sin^2 x} = \frac{\tan^2 x + 1 - 1}{\sin^2 x} = \frac{\tan^2 x}{\sin^2 x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{\sin^2 x}} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$(17) \tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x} = \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} = \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot (1 - \cos^2 x)$$

$$\tan^2 x (\sin^2 x + \cos^2 x - \cos^2 x) = \tan^2 x \cdot \sin^2 x$$

$$(18) \frac{1 - \sin^2 x}{\csc x - \sin x} = \sin x$$

$$\frac{\sin^2 x + \cos^2 x - \sin^2 x}{\frac{1}{\sin x} - \frac{\sin x}{1}} = \frac{\cos^2 x}{\frac{1 - \sin^2 x}{\sin x}} = \frac{\cos^2 x}{\frac{\sin^2 x + \cos^2 x - \sin^2 x}{\sin x}} = \frac{\cos^2 x}{\frac{\cos^2 x}{\sin x}} = \cos^2 x \cdot \frac{\sin x}{\cos^2 x} = \sin x$$

$$(19) \frac{\sec x - \cos x}{\cos x} = \tan^2 x$$

$$\frac{\sec x}{\cos x} - \frac{\cos x}{\cos x} = \frac{1}{\cos x} - 1 = \frac{1}{\cos x} \cdot \frac{1}{\cos x} - 1 = \frac{1}{\cos^2 x} - 1 = \sec^2 x - 1 = 1 + \tan^2 x - 1 = \tan^2 x$$

$$(20) \sin x (\csc x - \sin x) = \cos^2 x$$

$$\sin x \csc x - \sin^2 x = \sin x \cdot \frac{1}{\sin x} - \sin^2 x = 1 - \sin^2 x = \sin^2 x + \cos^2 x - \sin^2 x = \cos^2 x$$