

Sine and Cosine Graph Transformations

Fill in the tables.

	Period	Amplitude	Phase shift	Vertical Shift	Range
$y = -2\sin(\frac{1}{5}x - \pi)$	$\frac{2\pi}{\frac{1}{5}} = 10\pi$	2	5π	none	$[-2, 2]$
$y = \frac{1}{3}\cos(\theta + \frac{\pi}{3}) + 3$	$\frac{2\pi}{1} = 2\pi$	$\frac{1}{3}$	$-\frac{\pi}{3}$	3	$[2\frac{2}{3}, 3\frac{1}{3}]$
$y = \cos(3x - \frac{5\pi}{6}) - \frac{1}{2}$	$\frac{2\pi}{3}$	1	$\frac{5\pi}{18}$	$-\frac{1}{2}$	$[-\frac{1}{2}, \frac{1}{2}]$
$y = 5\cos(\frac{1}{2}x) + 4$	$\frac{2\pi}{\frac{1}{2}} = 4\pi$	5	none	4	$[-1, 9]$
$y = -4\sin(2\theta)$	$\frac{2\pi}{2} = \pi$	4	none	none	$[-4, 4]$
$y = \frac{1}{2}\cos(\theta + \frac{\pi}{2}) + 3$	$\frac{2\pi}{1} = 2\pi$	$\frac{1}{2}$	$-\frac{\pi}{2}$	3	$[2\frac{1}{2}, 3\frac{1}{2}]$
$y = -5\sin(\frac{1}{3}x) - \frac{1}{2}$	$\frac{2\pi}{\frac{1}{3}} = 6\pi$	5	none	$-\frac{1}{2}$	$[-5\frac{1}{2}, 4\frac{1}{2}]$
$y = \cos(\frac{3}{2}x) + \frac{5}{3}$	$\frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$	1	none	$\frac{5}{3}$	$[\frac{2}{3}, \frac{8}{3}]$

Write the sine equation for each of the following:

Amplitude	Period	Phase shift	Vertical Shift	Equation
2	$\frac{\pi}{2}$	$-\frac{\pi}{4}$	1	$y = 2\sin[4(x + \frac{\pi}{4})] + 1 \rightarrow y = 2\sin(4x + \pi) + 1$
4	3π	$\frac{\pi}{2}$	-4	$y = 4\sin[\frac{2}{3}(x - \frac{\pi}{2})] - 4 \rightarrow y = 4\sin(\frac{2}{3}x - \frac{\pi}{3}) - 4$
1	1	$\frac{\pi}{4}$	2	$y = \sin[2\pi(x - \frac{\pi}{4})] + 2 \rightarrow y = \sin(2\pi x - \frac{2\pi^2}{4}) + 2$
3	4	-1	$\frac{1}{2}$	$y = 3\sin[\frac{\pi}{2}(x + 1)] + \frac{1}{2} \rightarrow y = 3\sin(\frac{\pi}{2}x + \frac{\pi}{2}) + \frac{1}{2}$

Graph on a separate sheet of paper:

1. $y = 2\sin(4\theta - \frac{3\pi}{2}) + 1$	5. $y = 2\cos(\frac{1}{3}\pi\theta + 2\pi)$
2. $y = -\cos(\frac{\theta}{2} + \frac{5\pi}{3}) - 1$	6. $y = -2 + \sin(4\theta - \frac{2\pi}{3})$
3. $y = -3\sin(3\theta - 6) + 2$	7. $y = -4\sin(3\theta)$
4. $y = -1 + 4\cos(4\theta - \pi)$	8. $y = 2\cos(\frac{\theta}{4} + \frac{\pi}{6})$

① $y = 2 \sin \left(4\theta - \frac{3\pi}{2} \right) + 1 \Rightarrow y = 2 \sin \left(4 \left(\theta - \frac{3\pi}{8} \right) \right) + 1$

$A: 2$

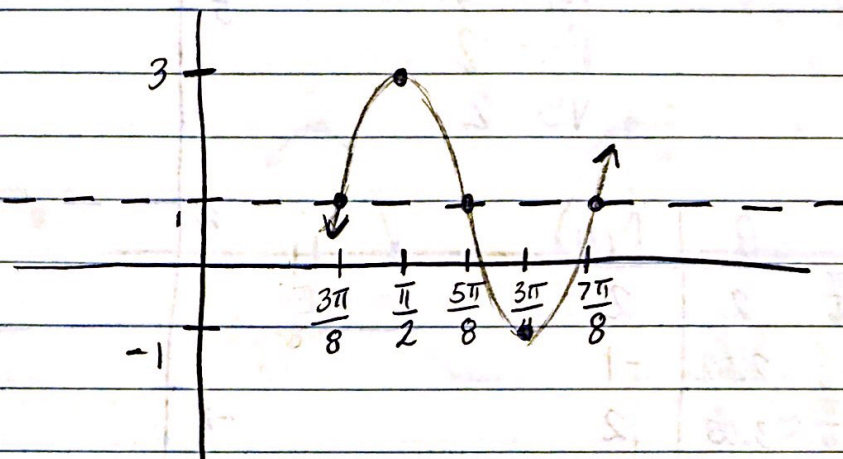
Per: $\frac{2\pi}{4} = \frac{\pi}{2}$

PS: $+\frac{3\pi}{8}$

VS: 1

$\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$

θ	$f(\theta)$
$\frac{3\pi}{8}$	1
$\frac{4\pi}{8} = \frac{\pi}{2}$	3
$\frac{5\pi}{8}$	1
$\frac{6\pi}{8} = \frac{3\pi}{4}$	-1
$\frac{7\pi}{8}$	1



② $y = -\cos \left(\frac{\theta}{2} + \frac{5\pi}{3} \right) - 1 \Rightarrow y = -\cos \left[\frac{1}{2} \left(\theta + \frac{10\pi}{3} \right) \right] - 1$

$A = 1$

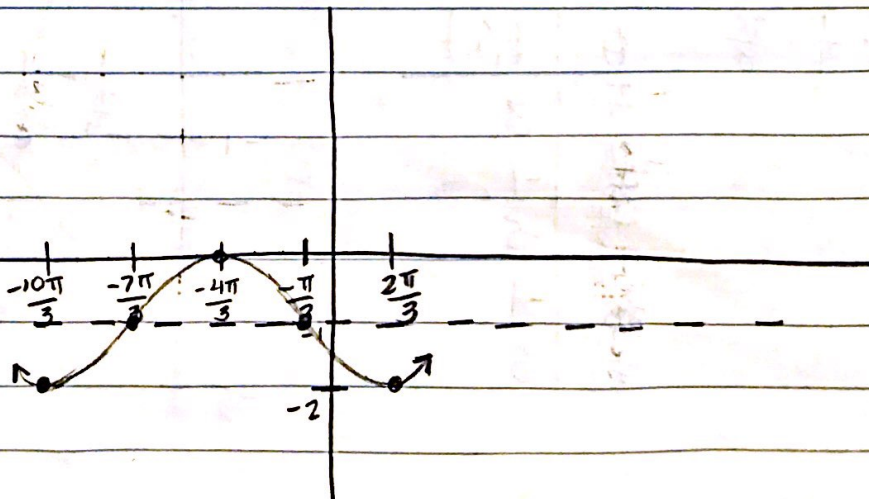
Per = $\frac{2\pi}{1/2} = 4\pi$

PS = $-\frac{10\pi}{3}$

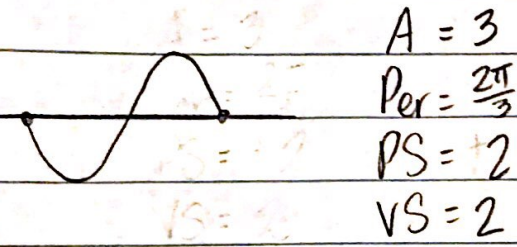
VS = -1

$4\pi \cdot \frac{1}{4} = \pi$

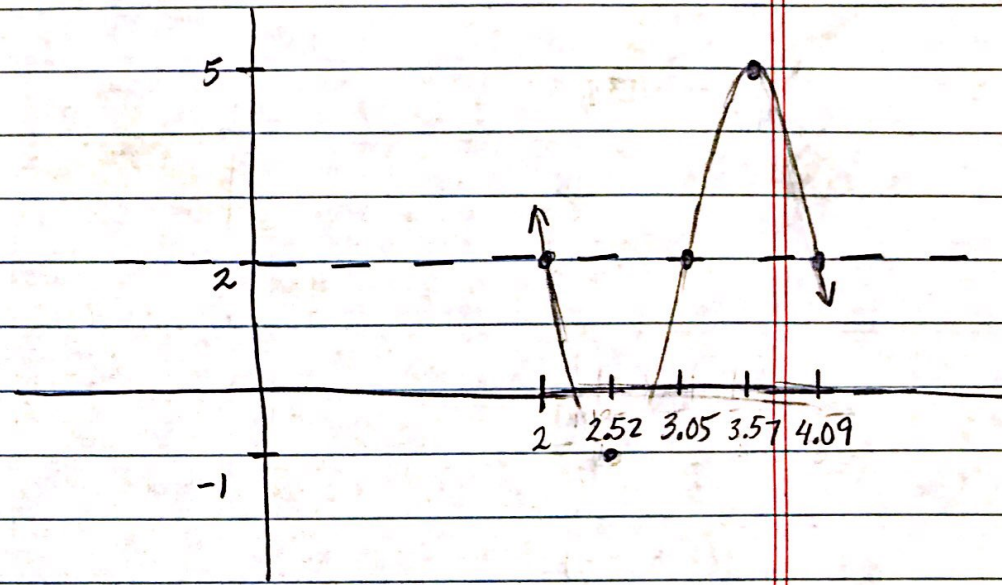
θ	$f(\theta)$
$-\frac{10\pi}{3}$	-2
$-\frac{7\pi}{3}$	-1
$-\frac{4\pi}{3}$	0
$-\frac{\pi}{3}$	-1
$\frac{2\pi}{3}$	-2



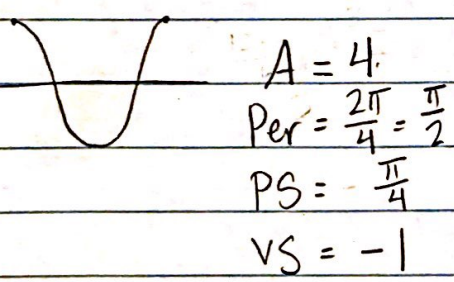
$$(3) \quad y = -3 \sin(3\theta - 6) + 2 \Rightarrow y = 3 \sin[3(\theta - 2)] + 2$$



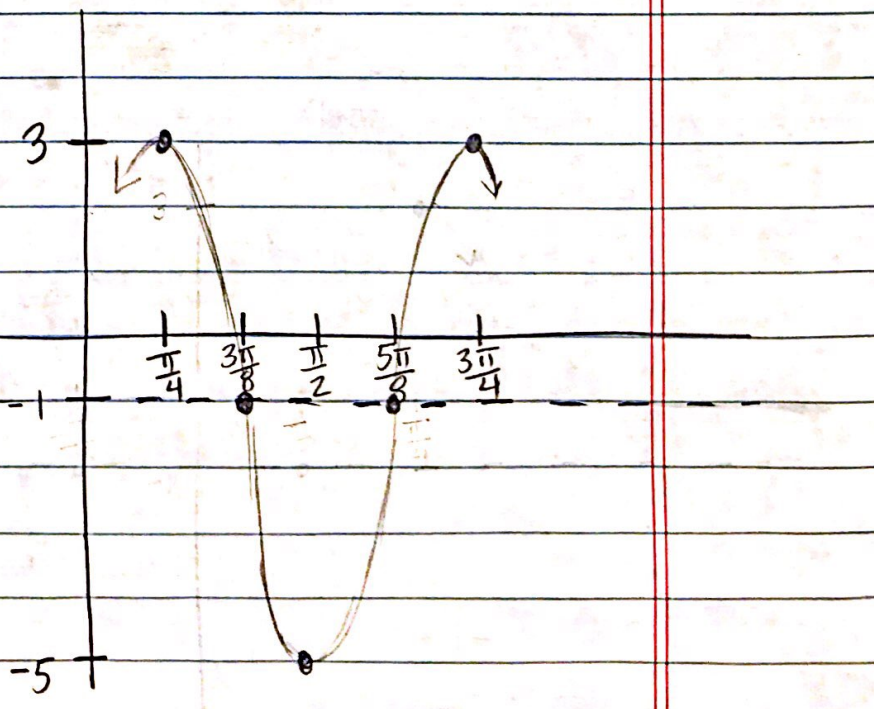
θ	$f(\theta)$
$\frac{2\pi}{3} \cdot \frac{1}{4} = \frac{\pi}{6}$	2
$2 + \frac{\pi}{6} \approx 2.52$	-1
$2 + \frac{\pi}{3} \approx 3.05$	2
$2 + \frac{\pi}{2} \approx 3.57$	5
$2 + \frac{2\pi}{3} \approx 4.09$	2



$$(4) \quad y = -1 + 4 \cos(4\theta - \pi) \Rightarrow y = -4 \cos[4(\theta - \frac{\pi}{4})] - 1$$



θ	$f(\theta)$
$\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$	3
$\frac{\pi}{4}$	-1
$\frac{3\pi}{8}$	-5
$\frac{\pi}{2}$	-1
$\frac{5\pi}{8}$	3
$\frac{3\pi}{4}$	3



$$(5) y = 2 \cos\left(\frac{1}{3}\pi\theta + 2\pi\right) \Rightarrow y = 2 \cos\left[\frac{1}{3}\pi(\theta + 6)\right]$$

$$A = 2$$

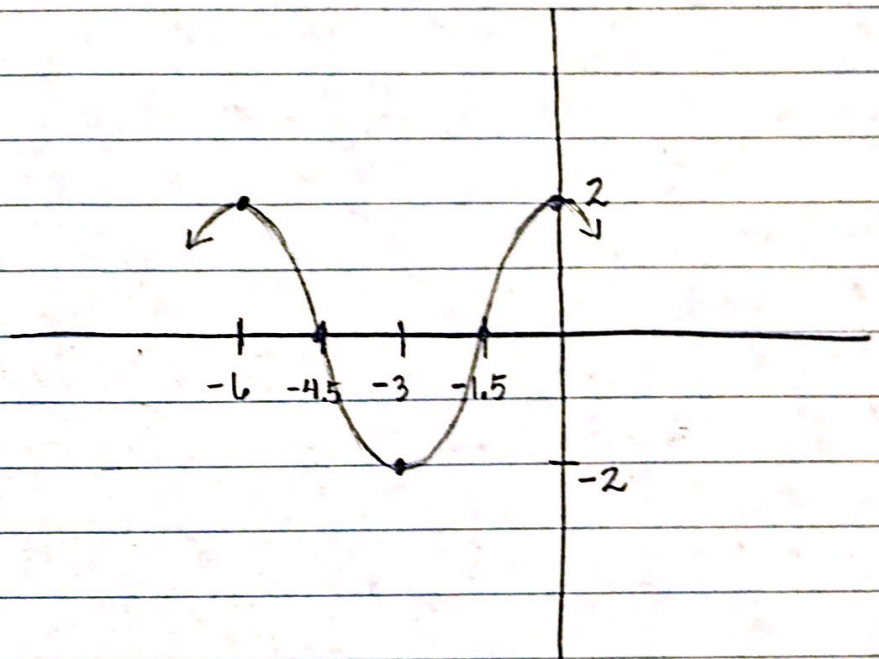
$$\text{Per} = \frac{2\pi}{\frac{1}{3}\pi} = 6$$

$$\text{PS} = -6$$

$$\text{VS} = \text{none}$$

$$6 \cdot \frac{1}{4} = \frac{3}{2}$$

θ	$f(\theta)$
-6	2
-4.5	0
-3	-2
-1.5	0
0	2



$$(6) y = -2 + \sin\left(4\theta - \frac{2\pi}{3}\right) \Rightarrow y = \sin\left[4\left(\theta - \frac{\pi}{6}\right)\right] - 2$$

$$A = 1$$

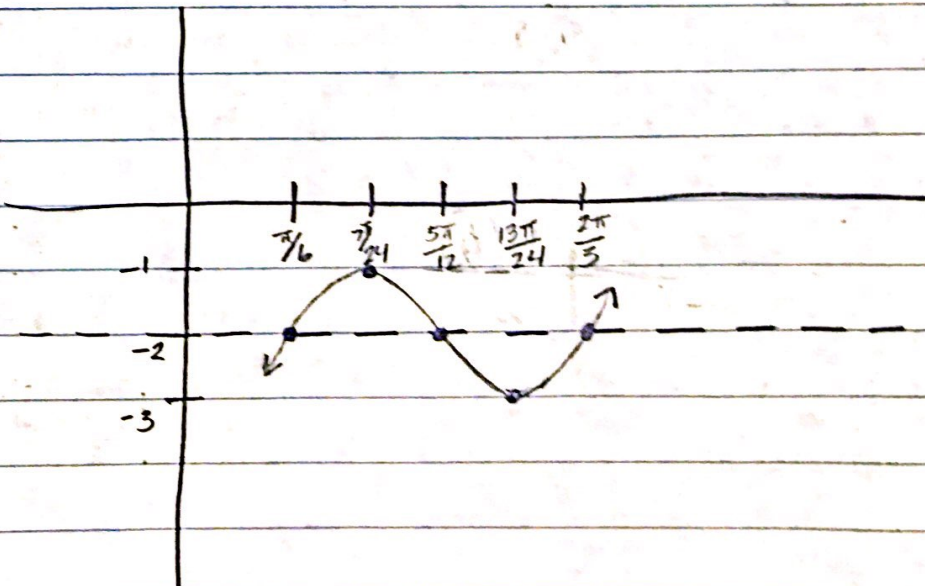
$$\text{Per} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\text{PS} = \frac{\pi}{6}$$

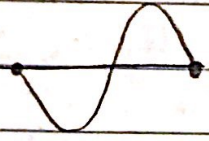
$$\text{VS} = -2$$

$$\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$$

θ	$f(\theta)$
$\frac{\pi}{6}$	-2
$\frac{7\pi}{24}$	-1
$\frac{5\pi}{12}$	-2
$\frac{13\pi}{24}$	-3
$\frac{2\pi}{3}$	-2



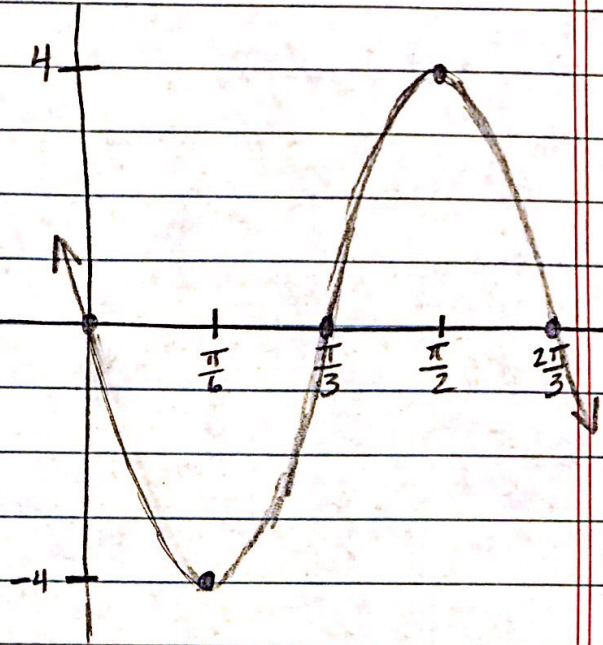
⑦ $y = -4 \sin(3\theta)$



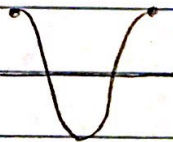
$A = 4$
 Per = $\frac{2\pi}{3}$
 PS = none
 VS = none

$\frac{2\pi}{3} \cdot \frac{1}{4} = \frac{\pi}{6}$

θ	$f(\theta)$
0	0
$\frac{\pi}{6}$	-4
$\frac{\pi}{3}$	0
$\frac{\pi}{2}$	4
$\frac{2\pi}{3}$	0



⑧ $y = 2 \cos\left(\frac{\theta}{4} + \frac{\pi}{6}\right) \Rightarrow y = 2 \cos\left[\frac{1}{4}\left(\theta + \frac{2\pi}{3}\right)\right]$



$A = 2$
 Per = $\frac{2\pi}{1/4} = 8\pi$
 PS = $-\frac{2\pi}{3}$
 VS = none

$8\pi \cdot \frac{1}{4} = 2\pi$

θ	$f(\theta)$
$-\frac{2\pi}{3}$	2
$\frac{4\pi}{3}$	0
$\frac{10\pi}{3}$	-2
$\frac{16\pi}{3}$	0
$\frac{22\pi}{3}$	2

