$$\underset{2\times 3}{\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}} \cdot \underset{3\times 2}{\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} R_1\cdot C_1 & R_1\cdot C_2 \\ R_2\cdot C_1 & R_2\cdot C_2 \end{bmatrix}}$$

Definition of Matrix Multiplication

If $A=\left[a_{ij}\right]$ is an $m \times n$ matrix and $B=\left[b_{ij}\right]$ is an $n \times p$ matrix, the product AB is an $m \times p$ matrix.

Finding the Product of Two Matrices

Example 7

Find the product AB where

$$A=\begin{bmatrix} -1 & 2 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \text{ and } B=\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 3+-8 & -2+2 \\ -12+8 & 8+-2 \\ -15+0 & 10+0 \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$$

3×2 $\quad 2\times 2$ $\quad 3\times 2$

Matrix Multiplication

Example 8

a. $2\times 3 \quad 3\times 3 \quad 2\times 3$

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 7 & -1 \\ -3 & 6 & 6 \end{bmatrix}$$

$$\begin{bmatrix} -2+0+-3 & 4+0+3 & 2+0+-3 \\ -4+-1+2 & 8+0+-2 & +4+0+2 \end{bmatrix}$$

b. $2\times 2 \quad 2\times 2 \quad 2\times 2$

$$\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

c. $$\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

d. $1\times 3 \quad 3\times 1 = 1\times 1$

$$\begin{bmatrix} 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 2+2+-3 \end{bmatrix}$$

e. $3\times 1 \quad 1\times 3 \quad 3\times 3$

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -4 & -6 \\ -1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix}$$

f. $3\times 2 \quad 3\times 4$

$$\begin{bmatrix} -2 & 1 \\ 1 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -2 & 3 & 1 & 4 \\ 0 & 1 & -1 & 2 \\ 2 & -1 & 0 & 1 \end{bmatrix}$$

undefined

$A \cdot B \neq B \cdot A$

Properties of Matrix Multiplication

Let A, B and C be matrices and let c be a scalar

1. $A(BC) = (AB)C$ — Associative Property of Matrix Mult.
2. $A(B + C) = AB + AC$ — Distributive Property
3. $(A + B)C = AC + BC$ — Distributive Property
4. $c(AB) = A(cB)$ — Associative Property of Scalar Mult.

Example 8: Application Problem

The number of touchdowns (TD), field goals (FG), point after touchdown (PAT), and two-point conversions (2EP) for the three top teams in a high school division for this season is shown in the table below. The other table shows the number of points each type of score is worth. Use the information to determine the team that scored the most points.

Score	Points
TD	6
FG	3
PAT	1
2EP	2

3x4

Team	TD	FG	PAT	2EP
Tigers	27	7	21	2
Rams	24	12	18	3
Eagles	21	14	12	9

4x1 3x1

$$\cdot \begin{bmatrix} 6 \\ 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 208 \\ 204 \\ \end{bmatrix}$$

$$\begin{bmatrix} 27\cdot 6 + 7\cdot 3 + 21\cdot 1 + 2\cdot 2 \\ 24\cdot 6 + 12\cdot 3 + 18\cdot 1 + 3\cdot 2 \\ 21\cdot 6 + 14\cdot 3 + 12\cdot 1 + 9\cdot 2 \end{bmatrix} = \begin{bmatrix} 208 \\ 204 \\ 198 \end{bmatrix} \leftarrow \text{Tigers}$$