

Equations of Circles

* Standard Form $(x-h)^2 + (y-k)^2 = r^2$
center = (h, k) radius = r

* General Form $Ax^2 + By^2 + Cx + Dy + E = 0$
 $A=1$ and $B=1$

ex: Write an equation: center $(3, -2)$ $r=4$
_{h k}

$$(x-3)^2 + (y-(-2))^2 = 4^2$$

Standard Form $(x-3)^2 + (y+2)^2 = 16$

Convert to general Form

$$(x-3)(x-3) + (y+2)(y+2) = 16$$

$$x^2 - 3x - 3x + 9 + y^2 + 2y + 2y + 4 = 16$$

General Form $x^2 + y^2 - 6x + 4y - 3 = 0$

ex: center $(-9, 0)$ $r = \sqrt{11}$

S.F. $(x-(-9))^2 + (y-0)^2 = (\sqrt{11})^2$

S.F. $(x+9)^2 + y^2 = 11$

ex: convert to S.F. & identify the center & radius

$$\text{GF: } x^2 + y^2 + 4x - 6y - 3 = 0$$

$$x^2 + 4x + y^2 - 6y = 3$$

$$\left(x^2 + 4x + \frac{4}{2}\right) + \left(y^2 - 6y + \frac{9}{2}\right) = 3 + 4 + 9$$

$$\text{SF: } (x+2)^2 + (y-3)^2 = 16$$

$$\text{center } (-2, 3) \quad r = 4$$

Steps

- ~~move~~ move the x terms together and y terms together
- move constant to the right
- complete the square (as needed) for x + y
- factor the left & simplify the right

ex: convert GF \rightarrow SF

$$\text{GF: } \frac{2x^2 - 16x + 2y^2}{2} = -20$$

$$x^2 - 8x + y^2 = -10$$

$$\left(x^2 - 8x + \frac{16}{2}\right) + y^2 = -10 + 16$$

$$\text{SF: } (x-4)^2 + y^2 = 6$$

$$\text{center } (4, 0) \quad r = \sqrt{6}$$

$$\text{distance formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}; \text{ midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$