Fundamental Trig Identities

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sin\theta = \frac{1}{\csc\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$

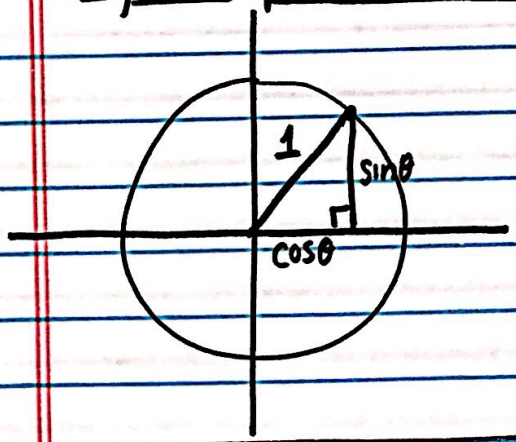
$\cot\theta = \frac{1}{\tan\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities



$\cos^2\theta + \sin^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$\cot^2\theta + 1 = \csc^2\theta$

scratch work

$* \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

$* \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\cot^2\theta + 1 = \csc^2\theta$

Simplifying Trig ~~Identities~~ Expressions

*no fractions, +, − in final answer

ex: $\sin x \cot x = \sin x \cdot \frac{\cos x}{\sin x} = \boxed{\cos x}$

ex: $(\csc^2\theta - 1)\sec^2\theta = (1 + \cot^2\theta - 1)\sec^2\theta =$
$\cot^2\theta \sec^2\theta = \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\cos^2\theta} = \frac{1}{\sin^2\theta} = \boxed{\csc^2\theta}$

ex: $\frac{\sec x}{\csc x} = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \boxed{\tan x}$

ex: $\frac{\cot^2\theta}{\cos^2\theta} = \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta}{1}} = \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\cos^2\theta} = \frac{1}{\sin^2\theta} = \boxed{\csc^2\theta}$

ex: $\sec x(1 - \sin^2 x) = \sec x(\sin^2 x + \cos^2 x - \sin^2 x)$
$= \sec x \cdot \cos^2 x = \frac{1}{\cos x} \cdot \frac{\cos^2 x}{1} = \frac{\cos^2 x}{\cos x} = \boxed{\cos x}$

ex: $\tan x(\cot x + \tan x) = \frac{\sin x}{\cos x}\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)$

$1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x = \boxed{\sec^2 x}$

ex: $\cot x - \cos^3 x \csc x = \frac{\cos x}{\sin x} - \frac{\cos^3 x}{1} \cdot \frac{1}{\sin x}$

$\frac{\cos x}{\sin x} - \frac{\cos^3 x}{\sin x} = \frac{\cos x - \cos^3 x}{\sin x} = \frac{\cos x(1 - \cos^2 x)}{\sin x}$

$\frac{\cos x(\sin^2 x + \cos^2 x - \cos^2 x)}{\sin x} = \frac{\cos x \cdot \sin^2 x}{\sin x}$

$= \boxed{\cos x \cdot \sin x}$

ex: $\csc\theta \sec\theta - \tan\theta = \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$\frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\sin\theta} = \frac{1 - \sin^2\theta}{\cos\theta\sin\theta} = \frac{\sin^2\theta + \cos^2\theta - \sin^2\theta}{\cos\theta\sin\theta}$

$\frac{\cos^2\theta}{\cos\theta\sin\theta} = \frac{\cos\theta}{\sin\theta} = \boxed{\cot\theta}$