

Simplifying Trigonometric Expressions

Simplify the following trigonometric expressions.

1. $\sec x \cos x$

$$\frac{1}{\cos x} \cdot \cos x = \boxed{1}$$

2. $\cot^2 x - \csc^2 x$

$$\frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x} = \frac{\cos^2 x - 1}{\sin^2 x} = \frac{\cos^2 x - (\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{\cos^2 x - \sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-\sin^2 x}{\sin^2 x} = \boxed{-1}$$

3. $\tan x \csc x$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = \frac{1}{\cos x} = \boxed{\sec x}$$

4. $(1 - \cos^2 x)(\csc x)$

$$(\sin^2 x + \cos^2 x - \cos^2 x) \cdot \frac{1}{\sin x} = \sin^2 x \cdot \frac{1}{\sin x} = \boxed{\sin x}$$

5. $\cos^2 x (\sec^2 x - 1)$

$$\cos^2 x (1 + \tan^2 x - 1) = \cos^2 x \cdot \tan^2 x = \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x} = \boxed{\sin^2 x}$$

6. $\frac{\csc x}{\sec x}$

$$= \frac{\frac{1}{\sin x}}{\frac{1}{\cos x}} = \frac{1}{\sin x} \cdot \frac{\cos x}{1} = \boxed{\cot x}$$

7. $\cot x \sec x = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin x} = \boxed{\csc x}$

8. $\frac{\sec^2 x - 1}{\sin^2 x} = \frac{1 + \tan^2 x - 1}{\sin^2 x} = \frac{\tan^2 x}{\sin^2 x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{1}} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$

9. $\sin x (\csc x - \sin x)$

$$\sin x \left(\frac{1}{\sin x} - \sin x \right) = \frac{\sin x}{\sin x} - \frac{\sin^2 x}{1} = 1 - \sin^2 x = \sin^2 x + \cos^2 x - \sin^2 x = \boxed{\cos^2 x}$$

10. $\frac{\cot x}{\csc x}$

$$\frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \frac{\cos x}{\sin x} \cdot \frac{\sin x}{1} = \boxed{\cos x}$$

11. $\sin x \sec x$

$$\sin x \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos x} = \boxed{\tan x}$$

12. $\csc x \tan x + \sec x$

$$\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} + \frac{1}{\cos x} = \frac{1}{\cos x} + \frac{1}{\cos x} = \frac{2}{\cos x} = \boxed{2 \sec x}$$

Answer Choices					
sec x	-1	1	sin x	csc x	tan x
sin^2 x	sec^2 x	cos^2 x	cos x	cot x	2 sec x

$$(30) \cot x - \csc^2 x \cot x = \frac{\sin^2 \cos x}{\sin^2 \sin x} - \frac{\cos x}{\sin^2 x} = \frac{\sin^2 \cos x - \cos x}{\sin^2 x} = \frac{\cos x (\sin^2 x - 1)}{\sin^2 x} = \frac{\cos x (\sin^2 x - (\sin^2 x + \cos^2 x))}{\sin^2 x} = \frac{\cos x \cdot \cos^2 x}{\sin^2 x} = -\cot^3 x$$

22. $\csc x \sec x - \tan x$	23. $\csc x - \cos x \cot x$
24. $\sec x \cot x - \sin x$	25. $\frac{\tan x + \sin x \sec x}{\csc x \tan x}$
26. $\frac{1 - \sin^2 x}{\csc^2 x - 1}$	27. $\frac{\csc x \cos x + \cot x}{\sec x \cot x}$
28. $\frac{\sec x \csc x - \tan x}{\sec x \csc x}$	29. $\frac{\sec^2 x}{\cot^2 x + 1}$
30. $\cot x - \csc^2 x \cot x$	31. $\cot x - \cos^3 x \csc x$

$$(31) \cot x - \cos^3 x \csc x = \frac{\cos x}{\sin x} - \frac{\cos^3 x}{\sin x} = \frac{\cos x - \cos^3 x}{\sin x} = \frac{\cos x (1 - \cos^2 x)}{\sin x} = \frac{\cos x (\sin^2 x + \cos^2 x - \cos^2 x)}{\sin x} = \frac{\cos x \sin^2 x}{\sin x} = \cos x \sin x$$

$$(22) \csc x \sec x - \tan x = \frac{1}{\sin x} \cdot \frac{1}{\cos x} - \frac{\sin x \cdot \sin x}{\cos x \sin x} = \frac{1 - \sin^2 x}{\sin x \cos x} = \frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \cot x$$

$$\frac{1 - \sin^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$(23) \csc x - \cos x \cot x = \frac{1}{\sin x} - \cos x \cdot \frac{\cos x}{\sin x} = \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x + \cos^2 x - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x$$

$$(24) \sec x \cot x - \sin x = \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} - \sin x = \frac{1}{\sin x} - \frac{\sin x \cdot \sin x}{\sin x} = \frac{1 - \sin^2 x}{\sin x} = \frac{\sin^2 x + \cos^2 x - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x} = \cos x \cot x$$

$$(25) \frac{\tan x + \sin x \sec x}{\csc x \tan x} = \frac{\tan x + \sin x \cdot \frac{1}{\cos x}}{\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}} = \frac{\tan x + \tan x}{\frac{1}{\cos x}} = \frac{2 \tan x}{\frac{1}{\cos x}} = \frac{2 \sin x}{\cos x} \cdot \frac{\cos x}{1} = 2 \sin x$$

$$(26) \frac{1 - \sin^2 x}{\csc^2 x - 1} = \frac{\sin^2 x + \cos^2 x - \sin^2 x}{1 + \cot^2 x - 1} = \frac{\cos^2 x}{\cot^2 x} = \frac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}} = \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x} = \sin^2 x$$

$$(27) \frac{\csc x \cos x + \cot x}{\sec x \cot x} = \frac{\frac{\cos x}{\sin x} \cdot \frac{\sin x}{1} + \frac{\cos x}{\sin x} \cdot \frac{\sin x}{1}}{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}} = \frac{\cos x + \cos x}{\frac{\sin x}{\cos^2 x}} = \frac{2 \cos x}{\frac{\sin x}{\cos^2 x}} = \frac{2 \cos x \cdot \cos^2 x}{\sin x} = 2 \cos^3 x$$

$$(28) \frac{\sec x \csc x - \tan x}{\sec x \csc x} = \frac{\sec x \csc x}{\sec x \csc x} - \frac{\tan x}{\sec x \csc x} = 1 - \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x \sin x}} = 1 - \frac{\sin x \cdot \cos x \cdot \sin x}{\cos x} = 1 - \sin^2 x = \cos^2 x$$

$$(29) \frac{\sec^2 x}{\cot^2 x + 1} = \frac{1}{\cos^2 x} = \frac{1}{\cos^2 x} \cdot \frac{1}{\frac{1}{\sin^2 x}} = \frac{1}{\cos^2 x} \cdot \sin^2 x = \tan^2 x$$