## Operations with Matrices

### Order of Matrices

R × C

Example 1: Determine the order of each Matrix

a. $[2]$ 1 × 1

b. $\begin{bmatrix} 1 & -3 & 0 & \frac{1}{2} \end{bmatrix}$ 1 × 4

c. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 2 × 2

d. $\begin{bmatrix} 5 & 0 \\ 2 & -2 \\ -7 & 4 \end{bmatrix}$ 3 × 2

### Equality of Matrices

Example: Solve for $a_{11}$, $a_{12}$, $a_{21}$, and $a_{22}$ in the following matrix equation.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

$a_{11} = 2$ $a_{12} = -1$

$a_{21} = -3$ $a_{22} = 0$

### Definition of Matrix Addition

If $A = [a_{ij}]$ and $B = [b_0]$ are matrices of order *m x n*, there is the *m x n* matrix

$$A + B = [a_{ij} + b_{ij}]$$

The sum of two matrices of different orders is undefined.

### Addition of Matrices

Example 2

a. $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ -1 & 3 \end{bmatrix}$

2 × 2 2 × 2 2 × 2

b. $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1-2 \\ 1 & 2 \quad 3 \end{bmatrix}$

2 × 3 2 × 3 2 × 3

c. $\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

3 × 1 3 × 1 3 × 1

d. The sum of A + B

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 3 & -2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$$

3 × 3 3 × 2

not possible

"undefined"

**Definition of Scalar Multiplication**

If $A=\left[a_{ij}\right]$ is an $m \times n$ matrix and $c$ is a scalar, the scalar multiple of $A$ by $c$ is the $m \times n$ matrix $cA=\left[ca_{ij}\right]$

**Scalar Multiplication and Matrix Subtraction**

Example 3: For the following matrices, find a) 3A, b) –B c) 3A – B

$$A=\begin{bmatrix}2 & 2 & 4\\ -3 & 0 & -1\\ 2 & 1 & 2\end{bmatrix} \qquad B=\begin{bmatrix}2 & 0 & 0\\ 1 & -4 & 3\\ -1 & 3 & 2\end{bmatrix}$$

*a.* $3A$

$$3\begin{bmatrix}2 & 2 & 4\\ -3 & 0 & -1\\ 2 & 1 & 2\end{bmatrix}=\begin{bmatrix}6 & 6 & 12\\ -9 & 0 & -3\\ 6 & 3 & 6\end{bmatrix}$$

*b.* $-B$

$$=\begin{bmatrix}-2 & 0 & 0\\ -1 & 4 & -3\\ 1 & -3 & -2\end{bmatrix}$$

*c.* $3A-B$

$$=\begin{bmatrix}4 & 6 & 12\\ -10 & 4 & -6\\ 7 & 0 & 4\end{bmatrix}$$

**Properties of Matrix Addition and Scalar Multiplication**

Let A, B, and C be m x n matrices and let c and d be scalars

| | |
|---|---|
| 1. $A + B = B + A$ | Commutative Property of Matrix Addition |
| 2. $A + (B + C) = (A + B) + C$ | Associative Property of Matrix Addition |
| 3. $(cd)A = c(dA)$ | Associative Property of Scalar Multiplication |
| 4. $IA = A$ | Scalar Identity |
| 5. $c(A + B) = cA + cB$ | Distributive Property |
| 6. $(c + d)A = cA + dA$ | Distributive Property |

**Addition of More than Two Matrices**

Example 4: Find the sum

$$\begin{bmatrix}1\\2\\-3\end{bmatrix}+\begin{bmatrix}-1\\-1\\2\end{bmatrix}+\begin{bmatrix}0\\1\\4\end{bmatrix}+\begin{bmatrix}2\\-3\\-2\end{bmatrix} = \begin{bmatrix}2\\-1\\1\end{bmatrix}$$

**Using the Distributive Property**

Example 5

Evaluate the expression $3\left(\begin{bmatrix}-2 & 0\\4 & 1\end{bmatrix}+\begin{bmatrix}4 & -2\\3 & 7\end{bmatrix}\right) = 3\begin{bmatrix}2 & -2\\7 & 8\end{bmatrix} = \begin{bmatrix}6 & -6\\21 & 24\end{bmatrix}$

**Solving a Matrix Equation**

Example 6a

Solve for X in the equation $3X + A = B$

$A=\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}$ and $B=\begin{bmatrix}-3 & 4\\2 & 1\end{bmatrix}$

$3X + \begin{bmatrix}1 & -2\\0 & 3\end{bmatrix} = \begin{bmatrix}-3 & 4\\2 & 1\end{bmatrix}$

$-\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix} \quad \begin{bmatrix}1 & -2\\0 & 3\end{bmatrix}$

$\frac{1}{3}\cdot 3X = \frac{1}{3}\cdot\begin{bmatrix}-4 & 6\\2 & -2\end{bmatrix}$

$X = \begin{bmatrix}-4/3 & 2\\2/3 & -2/3\end{bmatrix}$

Example 6b

Solve for x and y

$2\left(\begin{bmatrix}3x & -1\\8 & 5\end{bmatrix}+\begin{bmatrix}4 & 1\\-2 & -y\end{bmatrix}\right)=\begin{bmatrix}26 & 0\\12 & 8\end{bmatrix}$

$2\begin{bmatrix}3x+4 & 0\\6 & 5-y\end{bmatrix} = \begin{bmatrix}26 & 0\\12 & 8\end{bmatrix}$

$\begin{bmatrix}6x+8 & 0\\12 & 10-2y\end{bmatrix} = \begin{bmatrix}26 & 0\\12 & 8\end{bmatrix}$

$6x+8 = 26$ and $10-2y = 8$

$-8\ \ -8 \qquad -10\ \ -10$

$6x = 18 \qquad -2y = -2$

$x = 3$ and $y = +1$